THE JOB THAT WILL BE BORN

Generational cycle, URSS, and return on parental investment in Costa Rica, South Korea, and Italy

Original title (Portuguese): O emprego que nascerá: ciclo geracional, URSS e retorno do investimento parental em Costa Rica, Coreia do Sul e Itália

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ABSTRACT

We investigate whether societies whose N* index remains in the equilibrium zone can close a virtuous cycle among three generational cohorts (Descendants, Structurers and Ascendants). The premise is accounting-based: each additional birth is a new consumer, raises final demand and spontaneously generates work. The study covers Italy and South Korea (2024 N* of 0.61 and 0.48, collapse zone) and Costa Rica (1.32, equilibrium zone). N* is the square root of (Pop_Base ÷ Pop_Top) × (Births ÷ Deaths) × F, with F = TFR(t) ÷ TFR(t-25); it is therefore a demographic identity, not evidence, and the conclusions rest on variables that do not derive from it. Under the UN trend, the 2099 N* lies between 0.29 and 0.46. With fertility converging to 2.1 in 25 years (Target), it is 1.01 for Italy, 0.90 for South Korea (0.902, at the edge of the threshold) and 0.88 for Costa Rica, which starts in equilibrium and leaves it; the Target slows the deterioration but does not restore 2024 ratios. Preliminary input-output results (OECD ICIO 2019) indicate that each additional child sustains roughly 0.19 to 0.31 gross jobs per year (a demand ceiling), around 0.25 on average. A couple with 2.0 children thus generates about 0.5 jobs from children's demand alone and, because the parents remain in the Structurers cohort for much longer, the full cycle approaches, as an order of magnitude, 1.5 jobs. The cost of children falls mainly on parents (in Costa Rica, private transfers finance 61% of the Descendants' consumption); with that cost, the Structurers' labor income under the Target has a trough of -5% to -21% relative to the Trend and overtakes it only between 2067 and 2081. The return on investing in children is nil for parents or collective. Costa Rican literature identifies the gap: national accounts do not measure the time devoted to unpaid production, which is part of the real cost of caring for others (Jiménez-Fontana, 2015), and the care regime is socially unequal (Filgueira and Martínez Franzoni, 2019). Without return to the investing Sustainable Social Reproduction Units (URSS, from the Portuguese acronym), the burden stays with the Structurers and the three-link cycle weakens. Findings are conditional and exploratory.

Keywords: National Transfer Accounts; N*; generational cycle; URSS; fertility; employment; unpaid production; return on investment; intergenerational sustainability.

1. INTRODUCTION

Countries with very low fertility, such as Italy and South Korea, and countries that have reached this level more recently, such as Costa Rica, face a common dilemma through different trajectories: what happens to domestic demand, employment, and labor income when fewer children are born, and what changes if fertility approaches the level that sustains N* equilibrium?

The traditional approach is usually demographic, focusing on the dependency ratio and population aging. Here, the question is asked from the standpoint of consumption and the economic cycle. Every child born demands food, care, education, health care, and housing from the earliest years; this consumption supports domestic activity and employment, even when part of it is imported or displaces other spending. Preliminary calculations in this study indicate that this demand supports approximately 0.19–0.31 gross jobs per additional child per year (about 0.25 on average). A couple with two children generates roughly half a job from the children's demand alone. Because the parents remain in the Structurers cohort for a much longer period, the full cycle associated with that couple approaches 1.5 jobs in order of magnitude. When this circuit closes, it supports the employability of the next generation.

The study draws on generational economics (National Transfer Accounts, NTA; Lee and Mason, 2011; United Nations, 2013), which measures, by age, what people consume, what they produce, and how flows are redistributed across ages. On this basis, we use the Narayama Systemic Index (N*), proposed in the author's earlier work (Muarrek, 2026b), to establish each country's starting position, and the Sustainable Social Reproduction Unit (URSS) as an observable category independent of employment status. The URSS is the investing unit. The thesis is that the three-link cycle (Descendants, Structurers, and Ascendants) is more stable when N* remains in the equilibrium zone and part of the return on this investment flows back to those who made it. N* is a measurement hypothesis, not a definitive measure.

The N*_Obs published on AINU.Systems is the square root of (Pop_Base ÷ Pop_Top) × (Births ÷ Deaths) × F, where F = TFR(t) ÷ TFR(t−25). It does not currently incorporate transfers or fiscal capacity, because the component based on NTA has not yet been integrated. A methodological circularity must therefore be avoided: because N* is a function of fertility and age structure, its trajectory under TFR scenarios is a demographic identity, not evidence of well-being. This article's conclusions rely on variables that are not functions of N*: jobs, value added, labor income, and support and caregiver ratios.

The working hypothesis is that countries remaining outside the equilibrium zone for long periods experience a contraction in the Structurers cohort. This compresses the domestic demand base, reduces the jobs supported by that demand, and increases pressure on the capacity to care for Ascendants. Productivity gains and greater longevity may mask this dynamic for a time, but the three-link circuit closes sustainably only when there are enough births to keep N* near equilibrium and when the investment made by URSS receives some return. Otherwise, the burden remains concentrated on Structurers, the attractiveness of this role declines, and the cycle weakens. This is a proposition to be tested. Structurers' labor income is presented as demographic accounting (Section 6.3), and net employment has not yet been estimated.

Generational circuit and methodological circularity. The argument has two parts that must not be conflated. The first is the generational circuit: each cohort is supported by the preceding one and

expects to be supported by the next. Thus, a reduction in the children's link and an extension of the Ascendants' link compress the Structurers' link, which concentrates labor income and bears the costs at both ends. If this compression reduces the base supporting Descendants and Ascendants, it tends to reinforce itself. This is a hypothesis: Section 6 measures the initial movements (compression, income concentration, and cost), not the feedback loop. The second part is the methodological circularity described above. Replacement fertility at 2.1 is treated as a necessary condition for population stabilization; whether it is sufficient is an empirical question. Longevity gains, productivity gains, and forced income transfers may mask demographic decline without resolving it. The test therefore examines the channel through which employment emerges spontaneously from births.

Central question. Under what conditions can maintaining N* in the equilibrium zone, combined with recognition of URSS investment, allow consumption generated by additional births to become employment, labor income, and material well-being for all three cohorts over time? The working theme ("The Job That Will Be Born") unfolds into three questions: how much employment and labor income does the demand generated by an additional child support; when and how does that income return to the Structurers who invested; and how does each country's starting position in N* shape the possibility of completing the cycle?

Contributions and organization. (i) A comparative demographic exploration of three countries under a common fertility scenario, decomposing N* and identifying the variables that move it beyond TFR. (ii) An explicit account of how the composition of structural profiles, which defines age cutoffs, is determined and how much it matters. (iii) A first input-output calculation of the demand ceiling for jobs per unit of consumption and per additional child in the three countries. (iv) Measurement of the cost of children to parents and the return on this investment using NTA. (v) An evidence register with reading levels, distinguishing information read in source documents from information obtained through summaries or press coverage. Section 2 presents the framework; Section 3, the hypotheses; Section 4, data and methods; Section 5, demographic results; Section 6, preliminary economic results; Section 7, discussion; Section 8, limitations; and Section 9, final considerations.

2. THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 Generational Economics and National Transfer Accounts

NTA estimates consumption, labor income, saving, and public and private transfers by age, making the economic life cycle explicit: children and older people consume more than they produce, and adults finance the difference through transfers and markets (Lee and Mason, 2011; United Nations, 2013). Mason and Lee (2022) discuss how population change affects the global economy through channels such as the support ratio. We use NTA to connect births to consumption by cohort (CFCc) and, with input-output tables (Miller and Blair, 2009), to connect consumption to domestic activity and employment.

2.2 The Care Gap and Unpaid Production

The Latin American NTA literature already documents what is omitted from this monetary accounting. Rosero-Bixby (2011) applies NTA to five countries in the region (Brazil, Chile, Costa Rica, Mexico, and Uruguay) to examine the effects of population aging. Using the NTA

methodology, Jiménez-Fontana (2015) shows that national accounts do not measure the time devoted to unpaid production, which receives no explicit compensation and represents part of the real cost of caring for others; the opportunity cost of this time depends heavily on educational attainment. Filgueira and Martínez Franzoni (2019) show that the combination of high income inequality and the absence of universal care services produces a socially unequal regime in which the work and cost of social reproduction are concentrated in particular groups and families. For South Korea, Hwang and Yoo (2025) use administrative data from the national health insurance system and estimate that women's earnings fall by more than 41% after the first child, with no recovery within five years; the drop is driven mainly by exit from employment (44% lower employment in the fifth year), and the loss one year after childbirth rose from 40% (cohorts born in 1976–80) to 46% (1981–85). This is a discussion paper, not peer-reviewed, and it measures the individual cost of motherhood rather than the cost recorded by NTA. This article uses monetary accounts and therefore does not measure care time.

2.3 Life Cycle and Saving

The life-cycle hypothesis (Modigliani and Brumberg, 1954; Ando and Modigliani, 1963) predicts that consumption exceeds income in youth, income exceeds consumption during working age, and individuals dissave after retirement. Aggregate saving therefore depends on age structure, and the long-run sign of the effect of higher fertility on saving is indeterminate (H4).

2.4 Very Low Fertility and the Tempo Effect

The threshold of 1.3 children per woman is associated with so-called lowest-low fertility (Kohler, Billari, and Ortega, 2002), while the low-fertility trap hypothesis (Lutz, Skirbekk, and Testa, 2006) discusses mechanisms that reinforce it. Period fertility may be depressed by the postponement of motherhood, known as the tempo effect (Bongaarts and Feeney, 1998). ECLAC (2025) shows that, in Latin America, part of the recent decline in period TFR reflects this postponement and recommends interpreting TFR alongside mean age at childbearing, cohort fertility, and tempo adjustment. For South Korea, Hwang (2023) examines recent trends in cohort fertility, which serve as a counterpoint to the period rate. In this article, the UN period TFR is the model input, and the tempo effect is a source caveat rather than an adjustment.

2.5 N*, Cohorts, and the URSS

The three cohorts are Descendants (from birth to the first cutoff), Structurers (between the two cutoffs), and Ascendants (from the second cutoff onward). The cutoffs derive from a composition of A–E structural profiles (Muarrek, 2026b), with each country represented by a weighted average of two or three adjacent profiles, rounded to the nearest integer (half values rounded upward). The cutoffs for each profile (Descendants from age 0 to the first value; Ascendants from the second value onward) are: A, 16 and 44; B, 18 and 55; C, 21 and 60; D, 23 and 62; E, 25 and 65. The N* framework was applied to the Italian case in Muarrek (2026a). The URSS (Active, Fulfilled, or Inactive) is observable and independent of employment status. The ratios used are A/E (Ascendants ÷ Structurers) and E/80+ (Structurers per person aged 80 or older, a potential rather than actual caregiver). The N* equilibrium zone (Self-Sustaining Equilibrium Point, PEA) ranges from 0.90 to 1.40; below 0.71 is Narayama Collapse (PEC), and between 0.71 and 0.90 is Accelerated Tension. These zones are provisional: under the N* Quality, Decomposition, and Falsifiability Protocol

(PQDF-N*), the robustness test (F4) is not met; country rankings are stable, but the zone assigned to several countries changes with small choices.

Terminology note: URSS is retained as the original Portuguese acronym for Unidade de Reprodução Social Sustentável (Sustainable Social Reproduction Unit).

3. HYPOTHESES

The hypotheses below are propositions to be tested. The rejection thresholds for H1, H3, H5, and H8 will be set and preregistered before the economic estimation; this version does not set them.

H1 (Consumption). Each additional child increases Descendants' CFCc in accordance with the age-specific consumption profile, which serves as a basic premise of the model.

H2 (Jobs). Additional consumption supports gross jobs, while net employment depends on idle capacity and spending displacement. These factors modulate the magnitude of the effect without negating the increase in demand.

H3 (Labor income). Aggregate labor income among Structurers is higher under the Target than under the Trend only after the additional cohorts enter the working-age population (the J-curve). The trough reflects the cost borne by parents during the generation in which they invest in their children.

H4 (Saving). Under the life-cycle hypothesis, aggregate saving may decline in the short term; its long-term direction is indeterminate.

H5 (Support ratio). Ceteris paribus, the economic support ratio is higher under the Target than under the Trend in the long run.

H6 (Caregivers). The potential supply of caregivers depends on fertility, whereas demand (the population aged 80 or older) changes very little between scenarios through 2099.

H8 (Average condition; exploratory). With TFR maintained around 2.1, the age structure stabilizes rather than continuing to deteriorate, although at a level that may be worse than in 2024. The hypothesis is stratified by starting position. In this version's exploration, the clause stating that N* enters the equilibrium zone at a TFR of 2.1 holds for Italy (1.01), is at the threshold for South Korea (0.902), and does not hold for Costa Rica (0.88). Reformulating H8 (for example, with an equilibrium TFR above 2.1) is one of the next steps (Section 9).

4. DATA AND METHODS

4.1 Sample

Table 1 - Sample, N* position, and available data (2024 N*_Obs; AINU.Systems).

Country	N* 2024	Zone	Role in the study	NTA and national accounts data
Italy	0.61	Narayama Collapse	Pilot; complete cohort accounts	NTA 2008 (consumption, labor income, saving); SHIW microdata; ICIO
South Korea	0.48	Narayama Collapse	Pilot; stress scenario (extreme TFR)	KOSIS 2024 (consumption, labor income; saving only from 2000, from the NTA portal); ICIO
Costa Rica	1.32	Self-Sustaining Equilibrium Point	Third case; contrast with its own NTA	Own NTA (age-specific files for 1991, 2004, 2008, and 2013; 2018 not obtained); ICIO; MIP/SUT 2022 and ENIGH 2018 unavailable

Source: AINU.Systems (2024 N*_Obs), NTA, CCP-UCR, and OECD (data availability).

The three countries were selected after the demographic exploration; therefore, their selection is not independent of their N* values. The study does not estimate a statistical relationship between N* and outcomes, and with n = 3 the comparison is descriptive. Retaining each country in the sample depends on an audit of data coverage, quality, and comparability. Comparison with countries for which household budget microdata are available (Mexico, Argentina, Brazil, and the United States), as well as with the other AINU.Systems countries, is left for future work.

Note on N* values. The 2024 N* values used in this article (Italy 0.61, South Korea 0.48, and Costa Rica 1.32) are N*_Obs, calculated without the four adjustments in the falsifiability protocol (migration, inertia, policies, and underreporting). The independent thesis in which the index was proposed lists lower values for Italy (0.49) and South Korea (0.37), because it applies these adjustments (NGII_Puro) and, in South Korea's case, used the C80/D20 profile composition that preceded the one adopted here (Table 13).

4.2 Structural Profiles and Age Cutoffs

The profile composition for each country is the author's judgment, not a model output (Muarrek, 2026b), and is supported by declared proxies: (P1) the TFR range by source, comparing UN/ECLAC and the national statistical institute with the profiles' TFR ranges (C: 1.2–2.1; D and E: below 1.3); (P2) employment at ages 60–64, with a threshold to be set before examining the data; and (P3) longevity, treated as a sensitivity rather than a weight, because by construction it reduces N* by increasing Pop_Top. The adopted composition is Italy C30/D70, South Korea C30/D70, and Costa Rica C50/D50. South Korea was placed alongside Italy because of their similar longevity and South Korea's even lower birth rate; since C30/D70 and C50/D50 have the same cutoffs (22/61), N* does not change between the two. Alternative compositions are assessed in Section 5.5.

Table 2 - Profile composition and age cutoffs (Descendants, Structurers, and Ascendants).

Country	Profile composition	Descendants	Structurers	Ascendants
Italy	C 30% + D 70%	Ages 0–22	Ages 23–60	Age 61 or older
South Korea	C 30% + D 70%	Ages 0–22	Ages 23–60	Age 61 or older
Costa Rica	C 50% + D 50%	Ages 0–22	Ages 23–60	Age 61 or older

Source: AINU.Systems (profile composition defined by the author; Muarrek, 2026b).

Table 3 - TFR ranges by source (P1 proxy).

Country	TFR 2024, UN (WPP 2024)	TFR in UN projection	National source	Reading
Italy	1.21	1.27 (2034), 1.34 (2049), 1.48 (2099)	ISTAT: not read in this version	Below 1.3 in 2024; exceeds 1.3 in 2049
South Korea	0.73	0.86 (2034), 1.02 (2049), 1.29 (2099)	KOSTAT: not read in this version	Below 1.3 throughout the projection
Costa Rica	1.32 (ECLAC: crossed 2.1 in 2003 and 1.5 in 2020)	1.32 through 2034; 1.38 (2049); 1.49 (2099)	INEC: approximately 1.2 in 2023–2024 (press and UNA article; official PDF not read)	C/D boundary: 1.32 in UN data, approximately 1.2 in INEC

Source: UN WPP 2024 (medium variant); ECLAC (2025); national sources as indicated.

The TFR ranges for profiles D and E coincide (TFR below 1.3), so the TFR rule does not distinguish D from E; this choice is made by the author. The model uses a fixed composition over time, which does not capture, for example, Italy's TFR rising above 1.3 in 2049.

4.3 Demographic Model and Scenarios

The model is a cohort-component model by single year of age and sex, based on the medium variant of the UN projections (WPP 2024), with TFR as the only lever: mortality and migration follow the UN projection. The Trend follows the UN TFR. Under the Target, TFR converges linearly to 2.1 over 25 years (starting in 2024) and remains there. The exploration is exploratory and non-preregistered; it uses a single scenario, without an uncertainty interval. For Section 5.4, local adjustments were made to the model for additional migration of adults aged 20–39, fertility timing, and age cutoffs.

4.4 From Consumption to Employment

The analytical chain is: additional birth → ↑ CFCc (Descendants) → imported content (CIEc) + domestic value added (VADc) → gross jobs (EDPc) → net jobs, after spending displacement and supply constraints. Idle capacity, imported content, and displacement conditions modulate the magnitude of the net effect; they do not negate the increase in demand. VADc and employment per unit of final consumption come from the OECD international input-output tables (ICIO, 2019, 45 industries; Yamano et al., 2023) and ILOSTAT employment by ISIC section, using a uniform intensity within each section; the values are demand ceilings (0% displacement). Gross jobs per additional child are calculated as the average consumption of a Descendant (the NTA total-consumption profile over the UN population, ages 0–22, applied to total ICIO consumption) multiplied by the average employment intensity of consumption (Table 19). For Italy, the AINU.Systems Directory (the project's country fact-sheet and simulation database) also provides a value of 0.251 gross jobs per additional child per year, an estimated upper bound. Structurers' labor income (Section 6.3) uses the NTA age-specific labor-income profile (YL), the children's life-cycle deficit (consumption minus labor income) as the cost, and the model population. The decomposition of jobs by final-demand component and the rule for attributing exports are presented in Section 6.1.

4.5 Evidence Levels

Each data item has a reading level: L1, source document read; L2, search summary or press coverage. Data below L1 are identified as such and do not support conclusions. In particular, the national TFR figures for Italy, South Korea, and Costa Rica are still at L2 or have not been read, and the numerical 2018 NTA for Costa Rica was not obtained. For Costa Rica, CCP-UCR provides age-specific files for 1991, 2004, 2008, and 2013; the 2013 file (current colones) is the basis for

Table 16, and the 2008 file is used for sensitivity analysis. Table 16 indicates the year and source for each country.

4.6 Activity Proxy for the Ascendant Cutoff (P2)

Table 4 - Employment rate by age group (ILOSTAT, employment as a share of the working-age population) and assessment of the rule based on 50% of the working-age rate.

Country (year)	Working-age population, 35–54 (average)	Ages 55–59	Ages 60–64	Age 65 or older (aggregated group)	60–64 ÷ working-age	65+ ÷ working-age	Assessment under the 50% rule
Italy (2025)	77.3%	70.9%	50.4%	6.1%	65%	8%	Falls below 50% between ages 60–64 and 65+: cutoff at approximately age 65
South Korea (2025)	80.1%	77.0%	66.4%	40.7%	83%	51%	65+ remains at 51% (aggregated group): cutoff at age 65 or older, exact point unresolved
Costa Rica	No age-group file			Employment at age 65+: 17.0% (men) and 5.2% (women), July 2024 (ENAHO)			Proxy not calculated

Source: ILO, ILOSTAT (2025); ratios calculated from employment rates by age group.

The Ascendant cutoff (currently age 61 in all three countries) is a consequential choice: moving it from 60 to 65 raises 2099 N* by approximately 0.07 (Section 5.4). If longevity and activity at older ages increase, it is natural for the cutoff to shift and for equilibrium N* to adjust. But this degree of freedom is falsifiable only if the cutoff follows an observable measure fixed before examining its effect on N*. The observable adopted as a proxy (P2) is the employment rate by age group (ILOSTAT, employment as a share of the working-age population): the cutoff would be the age at which the rate falls below a fraction of the working-age rate (ages 35–54). The 2025 data are shown in Table 4. A 50% fraction is a candidate, and the threshold must be set and preregistered by the author; this version deliberately does not recalculate N* using the new cutoff. At 50%, the cutoff is approximately age 65 in Italy (ages 60–64: 65% of the working-age rate; age 65 or older: 8%) and age 65 or older in South Korea, where the aggregate 65+ group (51%) does not allow the exact point to be determined. In both countries, the current cutoff of age 61 is earlier than the observed activity threshold (50% in Italy and 66% in South Korea are still employed at ages 60–64). Costa Rica has no age-group breakdown in the available file; employment at age 65 or older (17.0% of men and 5.2% of women in July 2024; Espinoza-Herrera and Méndez-Castellanos, 2026) is an indication, not the proxy. Limitations: employment rates at older ages also reflect the statutory retirement age, and the 65+ age group is aggregated.

5. DEMOGRAPHIC RESULTS (EXPLORATORY)

The N* trajectory is a demographic identity, not evidence; the zones are provisional.

5.1 N* Trajectory

Table 5 - N* in 2024 and 2099 (Trend and Target).

Country	N* 2024	N* 2099 Trend	N* 2099 Target	Interpretation
Costa Rica	1.32	0.38	0.88	Leaves equilibrium (1.32 in 2024): falls to 0.80 in 2074 and settles at 0.88, in Accelerated Tension; better than the Trend (0.38)
Italy	0.61	0.46	1.01	Enters the equilibrium zone
South Korea	0.48	0.29	0.90	Better than the Trend; reaches 0.90 in 2099, at the edge of the equilibrium threshold; trough of 0.58 in 2037

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

The Target avoids the deepest collapse: Italy crosses into the equilibrium zone by 2099 (1.01); South Korea reaches the threshold (0.902); and Costa Rica, which starts in equilibrium, leaves that zone. N* equilibrium is a process that takes decades, and replacement fertility does not guarantee it.

Table 6 - Trajectory in 25-year cycles (2024 is identical under both scenarios). A/E = Ascendants ÷ Structurers.

Country	Scenario	N* 2049	N* 2074	N* 2099	A/E 2049	A/E 2074	A/E 2099
Costa Rica	Trend	0.75	0.44	0.38	0.62	1.07	1.20
Costa Rica	Target	1.31	0.80	0.88	0.61	0.90	0.78
Italy	Trend	0.46	0.45	0.46	1.01	1.04	1.06
Italy	Target	0.81	0.80	1.01	1.01	0.88	0.72
South Korea	Trend	0.36	0.26	0.29	1.06	1.53	1.40
South Korea	Target	0.93	0.62	0.90	1.05	1.14	0.75

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

5.2 Ascendants-to-Structurers Ratio

Table 7 - A/E ratio in 2024 and 2099.

Country	A/E 2024	A/E 2099 Trend	A/E 2099 Target	Target change vs. 2024
Costa Rica	0.31	1.20	0.78	+0.48
Italy	0.61	1.06	0.72	+0.11
South Korea	0.46	1.40	0.75	+0.29

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

In every case, the Target reduces deterioration relative to the Trend, but the 2099 ratio remains above its 2024 level: equilibrium stabilizes the system but does not restore the earlier ratio. Under the Target, the ratio peaks in 2049 in Italy (1.01) and in 2074 in South Korea (1.14) and Costa Rica (0.90), then declines through 2099 in all three (0.72, 0.75, and 0.78). Italy, already aged, peaks earlier; South Korea and Costa Rica, with large cohorts still aging, peak later. With only three countries, the stratification of H8 by starting position is descriptive.

5.3 Potential Caregivers

Table 8 - Structurers per person aged 80 or older (E/80+).

Country	E/80+ 2024	E/80+ 2099 Target	Relative decline
Costa Rica	20.6	2.8	-86%
Italy	6.4	3.1	-51%
South Korea	11.9	2.7	-78%

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

The decline is structural: most people who will be aged 80 or older in 2099 have already been born. The Target only mitigates the decline (exploratory support for H6). The absolute level in 2099 remains low in all three countries (between 2.7 and 3.1): even with replacement fertility, pressure on care remains high.

5.4 Why N* Does Not Remain in Equilibrium at a TFR of 2.1, and What Moves It

Costa Rica illustrates the mechanism. Under the Target, N* in 2049 (1.31) is supported by the inherited young age structure and by F = 1.59, which rewards the increase in TFR from 1.32 to 2.1. Once TFR is constant, F returns to 1.00 and the bonus disappears; at the same time, large cohorts reach age 60 (Pop_Top rises from 0.84 million in 2024 to 2.06 million in 2074), and N* falls to 0.80. By 2099, the population is stable and N* settles at 0.88: with a TFR of 2.1 and the UN mortality assumptions for 2099 (life expectancy of about 90 years), an approximate steady-state estimate is 0.87. The PEA lower bound (N_Base of 0.81) would require a Base ÷ Top ratio of about 0.81; with this longevity, replacement fertility yields about 0.76. Replacement fertility stabilizes the population but does not bring this N* measure into equilibrium.

Table 9 - Decomposition of Costa Rica's N* under the Target: (Pop_Base ÷ Pop_Top) × (Births ÷ Deaths) × F.

Year	TFR	Base ÷ Top	Births ÷ Deaths	F	N*	Zone
2024	1.32	1.84	1.80	0.53	1.32	Self-Sustaining Equilibrium Point
2049	2.10	0.85	1.27	1.59	1.31	Self-Sustaining Equilibrium Point
2074	2.10	0.68	0.95	1.00	0.80	Accelerated Tension
2099	2.10	0.79	0.99	1.00	0.88	Accelerated Tension

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

Table 10 - Costa Rica: minimum N* between 2030 and 2100 (and years with N* below 0.90), by target TFR and ramp duration.

Target TFR	25-year ramp	15-year ramp	10-year ramp
2.10	0.80 (36)	0.85 (30)	0.88 (13)
2.20	0.86 (17)	0.93 (0)	0.96 (0)
2.30	0.93 (0)	1.02 (0)	1.05 (0)
2.35	0.96 (0)	1.05 (0)	1.10 (0)

Source: AINU.Systems (demographic model with local adjustments; UN WPP 2024).

For N* never to fall below 0.90, the target must be about 2.35 with a 25-year ramp and about 2.2 with 10- or 15-year ramps. High targets move the index out of the zone: a target of 2.8 over 25 years takes 2099 N* to 1.72, in Population Tension. Only births in the coming years can change the Structurers cohort in 2074; the Ascendants in 2074 have already been born.

Table 11 - N* in 2099 and years with N* below 0.90 (2030–2100), by pathway, with TFR at 2.1.

Pathway	Costa Rica N* 2099	Costa Rica years < 0.90	South Korea N* 2099	South Korea years < 0.90
Base (TFR 2.1 in 25 years; UN migration)	0.88	36	0.90	67
Additional migration of +0.2% of the population per year (adults aged 20–39)	0.94	18	1.01	51
Additional migration of +0.4% of the population per year	0.99	0	1.09	37
Births 4 years earlier (fertility timing)	0.87	37	0.94	67
Ascendant cutoff at age 65 (definition, not a lever)	0.95	22	0.97	61

Source: AINU.Systems (demographic model with local adjustments; UN WPP 2024).

Additional migration of adults aged 20–39 is the pathway with the greatest effect: in Costa Rica, 0.4% of the population per year (about 21,000 people) keeps N* above 0.90 throughout, and the 2099 population reaches 8.2 million, compared with 5.6 million in the baseline. In South Korea, the 2037 trough (0.58) barely changes (at most 0.64), because the adults in that period have already been born. Fertility timing has little effect. Moving the Ascendant cutoff from 60 to 65 raises 2099 N* by about 0.07 in both countries, but this is a definitional choice, not a country-level policy lever. Additional migration is a sensitivity scenario, not a forecast. N* does not capture productivity, labor-force participation, or NTA: the economic content of the study is in Section 6.

5.5 Sensitivity to Structural Profile

Table 12 - Costa Rica: N* by profile composition (Target = TFR converging to 2.1 in 25 years; Trend = UN projection).

Composition	Cutoffs	N* 2024 (Trend)	N* 2049 (Trend)	N* 2099 (Trend)	N* 2024 (Target)	N* 2074 (Target)	N* 2099 (Target)
C100 (alternative)	21/60	1.25	0.72	0.37	1.25	0.77	0.85
C50/D50 (adopted)	22/61	1.32	0.75	0.38	1.32	0.80	0.88
D100	23/62	1.40	0.79	0.40	1.40	0.84	0.92

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

Table 13 - Italy and South Korea: N* by profile composition (Target = TFR converging to 2.1 in 25 years).

Country	Composition	Cutoffs	N* 2024	N* 2099 (Trend)	Minimum N* 2030–2100 (Target)	N* 2099 (Target)	Years < 0.90 (Target)
Italy	C50/D50 (alternative)	22/61	0.608	0.46	0.59	1.007	52
Italy	C30/D70 (adopted)	22/61	0.608	0.46	0.59	1.007	52
Italy	D100	23/62	0.640	0.48	0.63	1.049	50
South Korea	C80/D20 (alternative)	21/60	0.456	0.28	0.55	0.866	71
South Korea	C30/D70 (adopted; same cutoffs and N* as C50/D50)	22/61	0.484	0.29	0.58	0.902	67
South Korea	D100	23/62	0.516	0.30	0.61	0.940	63

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

Because the cutoffs are rounded, every mix between C70/D30 and C30/D70 produces the same cutoffs (22/61) and the same N*; the precise weight assigned to D between 30% and 70% does not change the result. In Italy, C30/D70 and C50/D50 do not change N*. In Costa Rica, replacing C100 with C50/D50 raises 2024 N* from 1.25 to 1.32 and 2099 Target N* from 0.85 to 0.88, without

changing the 2024 zone (equilibrium) or the conclusion that the Target does not return the index to equilibrium by 2099; even with D100, 2099 Target N* is 0.92, with 29 years below 0.90. In South Korea, C30/D70 and C50/D50 produce the same cutoffs and the same 2099 Target N* (0.90, at the equilibrium threshold); only D100 (cutoffs 23/62) would raise it to 0.94. The composition is fixed over time; testing "C evolving into D" would require a time-varying profile in the model.

6. PRELIMINARY ECONOMIC RESULTS

6.1 Value Added and Employment per Unit of Consumption (ICIO 2019)

Table 14 - Costa Rica: preliminary calculation using ICIO 2019, compared with Italy, South Korea, and Mexico (reference).

Indicator	Value	Note
Average VADc per US$ of final consumption	0.716	45 industries
Employment per US$ million of final consumption	25.7 persons	ILOSTAT + ICIO
Direct imported content (apparel and footwear)	≈ 49%	High
VADc Mexico (reference)	0.727	
Employment Mexico	34.0	
VADc Italy	0.725	
Employment Italy	9.3	
VADc South Korea	0.724	
Employment South Korea	15.1	

Source: OECD (ICIO 2019) and ILO (ILOSTAT).

Costa Rica has intermediate employment intensity: higher than Italy and South Korea and lower than Mexico (an out-of-sample reference). High imported content in some sectors (apparel and footwear, approximately 49% direct imported content) reduces VADc and potential domestic employment. These are demand ceilings (0% displacement), calculated using uniform employment intensity by ISIC section; they will be re-estimated using the BCCR national matrix (2022 base year, not yet accessible) and explicit cohort-allocation rules.

Table 15 - Total employment by final-demand component (ICIO 2019 and ILOSTAT) and attribution of exports to consumption.

Country (ICIO 2019)	Households	Government	Investment	Exports	Floor: consumption (households + government)	Ceiling: floor + exports financing consumption imports	Exports ÷ imports
Italy	40.2%	21.0%	14.0%	24.9%	61%	74%	1.09
South Korea	39.2%	16.5%	21.0%	23.4%	56%	64%	1.06
Costa Rica	44.9%	13.0%	11.1%	31.0%	58%	76%	1.08

Source: OECD (ICIO 2019) and ILO (ILOSTAT).

Interpretation and limitations (attribution). Total employment was decomposed by final-demand component (e′Ly, using the same employment intensity by section as the other estimates); the components add up to total employment (Table 15). Consumption (households plus government) supports 56%–61% of employment and is the floor of the "birth, consumption, employment" channel. Exports represent demand from residents of other countries, and a country exports, to a large extent, to pay for what it imports for its own consumption. Under balanced trade (the export-

to-import ratio ranges from 1.06 to 1.09), the share of exports proportional to imports embodied in consumption can be attributed to consumption, raising the ceiling to 64%–76%. This is an attribution rule, not a causal finding; investment (11%–21% of employment) was not attributed. Moreover, exports are not independent of demography: they depend on demand in other countries, and if trading partners also experience falling birth rates, the effect reaches the country through this channel. Exposure to trading partners' demographic trends was not measured in this version.

6.2 Operational Tables

Table 16 - Cohort Final Consumption (CFCc), derived from NTA: Italy (2008), South Korea (2024), and Costa Rica (2013).

Country	Reference year	Descendants	Structurers	Ascendants	Source / status
Italy	2008	€210.3 billion (19.4%)	€573.2 billion (52.8%)	€301.7 billion (27.8%)	NTA 2008 (total consumption, private and public; smoothed profile) × UN population (WPP 2024) in 2008; cutoffs 0–22, 23–60, 61+; total €1,085.2 billion; derived from the author's data files
South Korea	2024	KRW 324.6 trillion (21.5%)	KRW 823.7 trillion (54.6%)	KRW 360.9 trillion (23.9%)	KOSIS 2024 (total consumption, private and public; unsmoothed profile) × UN population (WPP 2024) in 2024; cutoffs 0–22, 23–60, 61+; total KRW 1,509.2 trillion; derived from the author's data files
Costa Rica	2013	CRC 5.92 trillion (31.5%)	CRC 10.56 trillion (56.2%)	CRC 2.32 trillion (12.3%)	CCP-UCR NTA (file CNT_CR2013; total consumption, current colones, age profile) × UN population (WPP 2024) in 2013; cutoffs 0–22, 23–60, 61+; total CRC 18.80 trillion; the sum matches the file's own total (0.2% difference)

Source: NTA (ntaccounts.org; KOSIS; CCP-UCR) and UN WPP 2024.

Table 17 - Cohort shares of population and consumption, and per capita consumption (Italy, South Korea, and Costa Rica).

Country (profile year)	Cohort	Ages	% of population	% of consumption	Per capita consumption
Italy (2008)	Descendants	0–22	22.0%	19.4%	€16,024
Italy (2008)	Structurers	23–60	53.2%	52.8%	€18,097
Italy (2008)	Ascendants	61–100	24.8%	27.8%	€20,433
South Korea (2024)	Descendants	0–22	18.0%	21.5%	KRW 34.8 million
South Korea (2024)	Structurers	23–60	56.1%	54.6%	KRW 28.4 million
South Korea (2024)	Ascendants	61–100	25.9%	23.9%	KRW 27.0 million
Costa Rica (2013)	Descendants	0–22	37.2%	31.5%	CRC 3.37 million
Costa Rica (2013)	Structurers	23–60	51.7%	56.2%	CRC 4.33 million
Costa Rica (2013)	Ascendants	61–100	11.1%	12.3%	CRC 4.45 million

Source: NTA (ntaccounts.org; KOSIS; CCP-UCR) and UN WPP 2024.

Interpretation and limitations. Total age-specific consumption profile (private and public; Italy: NTA, "Smooth Mean," ntaccounts.org portal; South Korea: KOSIS, unsmoothed) multiplied by the UN population by single year of age in the profile year, using the N* age cutoffs (Table 2). Values

are in local currency for the profile year, without deflation; therefore, only the structure (shares) can be compared across countries. In Italy (2008), Descendants account for 22.0% of the population and 19.4% of consumption, and their per capita consumption (€16,024) is below that of Structurers (€18,097) and Ascendants (€20,433). In South Korea (2019), Descendants account for 21.5% of the population and 24.5% of consumption, and their per capita consumption (KRW 24.3 million) is above that of Structurers (20.8) and Ascendants (19.7). In Costa Rica (2013, CCP-UCR NTA), Descendants account for 37.2% of the population and 31.5% of consumption, and their per capita consumption (CRC 3.37 million) is below that of Structurers (4.33) and Ascendants (4.45). The file separates public from private consumption: Descendants receive 47.9% of public consumption (education and health), compared with 26.6% of private consumption. The figures support the first link in H1 (Descendants' consumption has an independent weight), but do not measure the effect of an additional child. Limitations: smoothed profile (Italy and South Korea; not verified in the CCP-UCR file); profiles from different years; UN single-age population, which may differ from that used in NTA; the profile's terminal age (90+ in Italy; 85+ in South Korea; 91+ in Costa Rica) extended to older ages; and total consumption without separating goods and services.

Table 18 - VADc and employment per unit of consumption and per additional child (Descendants).

Country	VADc per US$ of consumption	Employment per US$ million	Gross jobs per additional child/year (ICIO + NTA)	Source
Italy	0.725	9.3	0.186 (Directory: 0.251)	ICIO 2019; Directory (EST nature; upper bound)
South Korea	0.724	15.1	0.314	ICIO 2019; KOSIS 2024 profile
Costa Rica	0.716	25.7	0.202 (range 0.184 to 0.202)	ICIO 2019; 0–22 ÷ average ratio from NTA 2013 (CCP-UCR)

Source: OECD (ICIO 2019), ILO (ILOSTAT), NTA, KOSIS, and AINU.Systems Directory.

Table 19 - Gross and net employment under different levels of displacement (per additional child per year; demand ceiling).

Country (method)	Average annual consumption per Descendant	Gross jobs per child/year (0% displacement)	50% displacement	100% displacement	Note
Italy (Directory)	€23,396 (2022)	0.251	0.133	0.0155	Directory (EST), national table; upper bound, without the child's future production
Italy (ICIO + NTA)	US$20,045 (2019)	0.186	0.093	not calculated	method test applied to the three countries: 26% below the Directory
South Korea (ICIO + NTA)	US$20,804 (2019)	0.314	0.157	not calculated	KOSIS 2024 profile; a Descendant's consumption is 19% above the average
Costa Rica (ICIO + NTA)	US$7,869 (2019)	0.202	0.101	not calculated	0–22 ÷ average ratio = 0.85 (NTA 2013, CCP-UCR); with the 2008 profile (0.77): 0.184

Source: OECD (ICIO 2019), ILO (ILOSTAT), NTA, KOSIS, and AINU.Systems Directory.

Interpretation and limitations (employment per child). Average consumption of one Descendant (NTA total-consumption profile over the UN population, ages 0–22) applied to total ICIO 2019 consumption and multiplied by the average employment intensity of consumption (employed persons per US$ million, direct and indirect, domestic). The result is 0.19 in Italy, 0.31 in South

Korea, and 0.20 in Costa Rica. South Korea leads for two reasons: a Descendant's consumption is 19% higher than the national average, and each US$ million of consumption supports more employment (15.1, compared with 9.3 in Italy). Costa Rica has the highest intensity (25.7), but consumption per Descendant (US$7,869) is about two-fifths of Italy's. As a method check, Italy yields 0.19 compared with 0.251 in the Directory (26% lower); therefore, the three values should be compared with one another, not with the Directory value. In Costa Rica, a Descendant's consumption is 85% of the national average in the 2013 CCP-UCR profile and 77% in the 2008 profile, which yields 0.184; replacing it with the profiles for Brazil, Mexico, and Argentina would yield 0.165–0.192. Limitations: average rather than marginal consumption (no household economies of scale); employment measured in employed persons, formal and informal, not full-time equivalents; uniform intensity by ISIC section; 2019 ICIO rather than national input-output matrices; no future production by the child or induced effects. The 50% displacement case is proportional; the 100% case was not calculated outside Italy.

Couple-level interpretation: the logic of the full cycle. The couple-level interpretation is the author's accounting exercise, constructed in five steps from the results in Table 19.

- **Step 1: the demand flow generated by one child.** Each additional child raises consumption in the economy from birth, and this consumption supports activity and employment in other sectors (Miller and Blair, 2009). Across the three cases, the impact averages about 0.25 gross jobs per child per year (the simple average of the ICIO + NTA method values, 0.19, 0.31, and 0.20, is 0.23). A couple with two children therefore supports about 0.5 jobs per year while the children are in the Descendants cohort.
- **Step 2: the lag between cohorts.** The parents' phase in the Structurers cohort is longer than the children's phase as Descendants. Under this study's cutoffs, Descendants span 23 years (ages 0–22) and Structurers 38 years (ages 23–60). As a simplification, the parents' phase is treated as approximately twice as long as the children's phase.
- **Step 3: the flow continues during the parents' phase.** If the flow of 0.5 jobs per year continues while the parents remain in the cohort, the equivalent for this phase is $0.5 \times 2 = 1.0$.
- **Step 4: summing the cycle.** The full cycle associated with the couple adds the children's phase and the parents' phase: $0.5 + 1.0 = 1.5$ jobs, measured in job-years over the cycle, not simultaneous jobs. Using the exact ratio between the two phases ($38 \div 23$, approximately 1.65), the result is $0.5 + 0.5 \times 1.65$, or approximately 1.3. The interpretation is therefore an order of magnitude between 1.3 and 1.5.
- **Step 5: sufficiency condition.** Approximately one and a half jobs per couple is equivalent to one full-time job plus one part-time job, which is compatible with the usual division of care work, in which mothers devote more time to children (Budig and England, 2001; Folbre, 2008). This sufficiency holds only if the investing URSS receives part of the return on its investment; otherwise, the burden remains with Structurers (Section 6.6). Moreover, jobs created by children's demand arise throughout the economy, and whether they reach the parents depends on that return.

Theoretical basis of the logic. Each link in the reasoning has support in the literature, although the calculation itself is the author's. (a) Demand as a determinant of employment goes back to Keynes (1937) and Hansen (1939), who associated population decline with falling investment and demand. (b) The cost of raising children is private while the benefit is collective: Folbre (1994) treats children as public goods, and Sinn (2004) shows that pay-as-you-go pensions partly decouple old-

age income from investment in one's own children, reducing private incentives to invest in them. (c) Consumption and income by age, which connect birth to demand and parents' labor, come from NTA (Lee and Mason, 2011; United Nations, 2013). (d) Intergenerational dependence in sustaining the economic system is the principle underlying overlapping-generations models (Samuelson, 1958) and the direction of wealth flows between generations (Caldwell, 1976). (e) Unpaid care and its unequal distribution are excluded from monetary accounts (Rosero-Bixby, 2011; Jiménez-Fontana, 2015; Filgueira and Martínez Franzoni, 2019).

Limit of the argument. This interpretation describes a demand channel, not a guaranteed net effect. Employment is also constrained by labor supply, and children will later become workers and consumers, which calls for caution about the idea of a fixed number of jobs; Acemoglu and Restrepo (2017), for example, find no negative relationship between aging and growth. The figure of 1.5 jobs per couple is the author's accounting illustration, not an estimated result.

The Italy Directory reports 0.251 gross jobs per additional child per year, 0.133 with 50% displacement, and 0.0155 with 100% displacement, an upper bound that does not include the child's future production; the 0.2–0.3 range applies to six countries in the Directory. These figures do not estimate saving (H4) or the economic support ratio (H5); labor income (H3) is addressed in Section 6.3.

Table 20 - Key demographic ratios (summary).

Country	A/E 2024	A/E 2099 Target	E/80+ 2024	E/80+ 2099 Target	N* 2099 Target
Costa Rica	0.31	0.78	20.6	2.8	0.88
Italy	0.61	0.72	6.4	3.1	1.01
South Korea	0.46	0.75	11.9	2.7	0.90

Source: AINU.Systems (demographic model, UN WPP 2024, medium variant).

6.3 Structurers' Labor Income (H3), in Demographic Accounting

Table 21 - Aggregate labor income of Structurers (ages 23–60), Target and Trend, with labor income by age held constant (2024 = 100 under the Trend).

Country (labor-income profile)	Target, index with 2024 = 100 (2049 / 2074 / 2099)	Trend, index (2049 / 2074 / 2099)	Target above Trend (2074 / 2099)	Difference reaches 1% / 5% / 10% in	Target minimum (year)
Italy (YL 2008)	73 / 65 / 73	73 / 56 / 47	15% / 54%	2056 / 2065 / 2070	65 (2072)
South Korea (YL 2024)	68 / 50 / 55	68 / 38 / 27	31% / 104%	2055 / 2061 / 2065	50 (2074)
Costa Rica (YL 2013)	97 / 82 / 85	97 / 69 / 52	18% / 62%	2055 / 2063 / 2068	82 (2079)

Source: NTA and KOSIS (YL variable) and AINU.Systems (demographic model, UN WPP 2024).

Interpretation and limitations (H3). Labor income per person by age (NTA, YL variable; Costa Rica 2013, Italy 2008, South Korea 2024) is held constant in real terms and multiplied by the model population aged 23–60; only population changes between scenarios. First, the Target and Trend coincide until about 2048, when the first additional cohort, born in 2025, reaches age 23; the difference reaches 1% in 2055–2056, 5% in 2061–2065, and 10% in 2065–2070. This timing is a demographic identity, not evidence. Second, without the cost of children there is no initial decline: the difference is never negative, so the downward arm of the H3 J-curve is missing; Table 22 adds this cost, measured by children's life-cycle deficit. Third, the Target does not restore the 2024 level: the minimum Structurers' labor income under the Target is 65% of the 2024 level in Italy (2072),

50% in South Korea (2074), and 82% in Costa Rica (2079), and in 2099 it stands at 73%, 55%, and 85%; under the Trend it reaches 47%, 27%, and 52%. Structurers account for 87%–91% of the entire population's labor income in 2024. Limitations: H3 thresholds were not preregistered, so this is not a test; labor income by age is held constant (no productivity, participation, or unemployment changes; the effect of number of children on parents' income enters only in Table 22 through the children's life-cycle deficit); profiles from different years and methodologies; model population (UN WPP 2024) and a single scenario.

Table 22 - Structurers' labor income under the Target minus the Trend, net of the cost of additional children (life-cycle deficit, ages 0–22), with and without demand return.

Country	Average cost of one child ÷ Structurer's income	Without demand return: trough (year); turns positive in	With demand return (ceiling): trough (year); turns positive in	Demand return ÷ cost	Net difference in 2099 (without / with return)
Italy	62%	-10.3% (2059); 2078	-5.1% (2055); 2069	45%	22% / 36%
South Korea	83%	-20.9% (2060); 2081	-9.8% (2057); 2069	46%	30% / 64%
Costa Rica	60%	-8.8% (2057); 2075	-4.7% (2054); 2067	40%	28% / 42%

Source: NTA, KOSIS, OECD (ICIO 2019), ILO (ILOSTAT), and AINU.Systems (demographic model).

Interpretation and limitations (cost of children). The cost of additional children is measured, not assumed: the age-specific life-cycle deficit (consumption minus labor income, NTA) from ages 0–22 is multiplied by the additional Target population at each age and charged entirely to Structurers (an upper bound: part is borne by Ascendants and working-age adults). On average, one child costs 60%–83% of a Structurer's annual income for 23 years. Without demand return, the trough is −10.3% in Italy (2059), −20.9% in South Korea (2060), and −8.8% in Costa Rica (2057), and the difference turns positive only in 2075–2081, after 50–56 negative years. Demand return (the spending on the child becomes someone's income) is estimated as jobs per child from Table 19 multiplied by labor income per job, and equals 40%–46% of the cost; included as an upper bound, the trough is −5.1% in Italy (2055), −9.8% in South Korea (2057), and −4.7% in Costa Rica (2054), with recovery in 2067–2069. In 2099, the Target is 22%–30% above the Trend (without return) or 36%–64% (with return). Two results follow: the J-curve exists, is deep, and lasts more than four decades; and its depth depends on how much of the cost is offset by demand, which is an upper bound here (it assumes idle capacity and that the jobs generated go to Structurers). This calculation replaces the free parameter in the previous version, whose cost for ages 0–5 underestimated the duration of the burden. Sensitivity for South Korea: with the 2019 profile from the NTA portal (the previous version), the paper obtained 0.30 jobs per child, a cost of 79% of a Structurer's income, and a trough of −19.7% in 2059 (recovery in 2077); with the revised KOSIS 2019 series the results barely change (0.29; 78%; −19.7%; 2077), so the difference from the 2024 profile comes from the year, not from the revision of the series. Limitations: constant labor income by age; no productivity change; the entire cost charged to Structurers; demand return is gross; profiles from different years.

6.4 Who Pays: Cohort Consumption Financing and Measured Burden (Costa Rica)

Table 23 - Financing of each cohort's consumption (Costa Rica, NTA 2013, CCP-UCR; % of cohort consumption).

Cohort (Costa Rica, 2013)	Consumption (CRC trillion)	Labor income	Net public transfers	Net private transfers	Asset reallocations
Descendants	5.92	9.2%	26.3%	61.4%	3.1%
Structurers	10.56	116.6%	-25.0%	-31.7%	40.1%
Ascendants	2.32	26.2%	54.4%	-11.0%	30.4%

Source: CCP-UCR (CNT_CR2013).

Table 24 - Costa Rica's J-curve with the NTA-measured burden (Structurers' labor income, Target minus Trend, net of cost).

Cost considered (Costa Rica, NTA 2013)	Trough (% of Structurers' income under Trend; year)	Turns positive in	Years with negative difference	Net difference in 2074 / 2099
Net private transfers to children only	-5.4% (2055)	2069	44	6% / 39%
Private plus the public share paid by Structurers (80%)	-7.9% (2057)	2073	48	1% / 31%
Total life-cycle deficit of children, without demand return	-8.8% (2057)	2075	50	0% / 28%
Total deficit, with demand return (ceiling)	-4.7% (2054)	2067	42	7% / 42%

Source: CCP-UCR (CNT_CR2013) and AINU.Systems (demographic model).

Interpretation and limitations (burden). In Costa Rica (2013 NTA, CCP-UCR), Descendants' consumption is financed by private transfers (61.4%), public transfers (26.3%), their own labor income (9.2%), and assets (3.1%) (Table 23). Structurers' labor income equals 116.6% of their consumption, and they transfer the surplus: 31.7% of consumption in net private transfers and 25.0% in public transfers, equivalent to 27% and 21% of their labor income; Ascendants finance 54.4% of consumption through public transfers (pensions and health care). Per child aged 0–22, the private transfer received is CRC 2.07 million per year, 41% of a Structurer's income (about 20% per adult in a couple), and the public share borne by Structurers (80% of public transfers paid) adds another 14%. The CCP-UCR 2004 report (Rosero-Bixby, Zúñiga-Brenes, and Collado, 2011) shows the same structure (private transfers equal 73.2% of consumption at ages 0–19; public transfers equal 50.4% of consumption at ages 65 or older). This measures an asymmetry: the cost of children is private and concentrated on parents, while the benefit (future workers and taxpayers) is collective; NTA shows the cost structure, not the reasons for fertility decisions. With this burden, rather than the total deficit, Costa Rica's J-curve trough is −5.4% (2055) with only the private share and −7.9% (2057) with the public share, with recovery in 2069 and 2073 (Table 24), without demand return. Only Costa Rica has the transfer profile available in digital files; for Italy and South Korea, the cost is the life-cycle deficit.

Literature and real cost. Time devoted to unpaid production is not measured by national accounts and is part of the real cost of caring for others (Jiménez-Fontana, 2015), meaning that families' actual effort is greater than the monetary record in NTA. This interpretation is consistent with Filgueira and Martínez Franzoni (2019), who show the unequal distribution of the care burden. These findings come from the cited literature and were not re-estimated in this article, which does not quantify unpaid production.

6.5 Cost Gradient by Number of Children and Falsifiability Test

Table 25 - Reference for rising costs with the number of children (linear extrapolation of average NTA cost; not actual expenditure).

Cost basis	Children	Cumulative cost (years of a Structurer's income)	Annual peak (% of couple's income)	% of couple's labor income over working life
Costa Rica, private transfer (linear)	1	9.4	25%	12%
	2	18.7	50%	25%
	3	28.1	74%	37%
	4	37.4	96%	49%
Costa Rica, private, income at parents' ages and square-root scale	1	9.4	22%	14%
	2	17.2	40%	25%
	3	24.1	55%	35%
	4	30.3	67%	44%
Costa Rica, private + 80% of public (linear)	1	12.6	33%	17%
	2	25.2	67%	33%
	3	37.8	99%	50%
	4	50.4	131%	66%
Italy, total life-cycle deficit (linear)	1	14.3	41%	19%
	2	28.5	80%	38%
	3	42.8	116%	56%
	4	57.1	151%	75%
South Korea, total life-cycle deficit (linear)	1	19.1	60%	25%
	2	38.3	116%	50%
	3	57.4	171%	76%
	4	76.6	224%	101%

Source: NTA, KOSIS, and CCP-UCR; linear extrapolation of average cost (reference, not expenditure).

Interpretation and limitations (gradient). To establish a reference for rising costs with the number of children, the average child cost by age (NTA) was multiplied by n children spaced three years apart and compared with the couple's labor income, twice the average income of a Structurer (Table 25). In Costa Rica, using the private share, one child costs 9.4 years of one adult's income, or 12% of the couple's labor income over their working lives; four children, 49%, with an annual peak of 96% of the couple's income. In Italy and South Korea, using the total life-cycle deficit (private and public), four children reach 75% and 101%. No couple allocates this share to children: the scale does not measure actual expenditure. This is a falsifiability test of the linear extrapolation, and it fails. The average datum is nevertheless coherent: in Costa Rica, Structurers transfer 27% of labor income in net private transfers, with 0.72 children aged 0–22 per Structurer; what does not hold is multiplying the average cost by three or four. Using the couple's income at parental ages and

economies of scale based on the square root of household size, the peak for four children falls from 96% to 67% and the lifetime cost from 49% to 44%, still high values. There are three possible explanations: lower consumption per child in large families (chronic deprivation); assistance from third parties (the state, extended family, remittances); and higher income in these families (more working adults, older children). The NTA profiles do not distinguish among them. There is an indication that the cost attributed exceeds actual expenditure: in the Italy Directory, the ratio of consumption at ages 0–24 to ages 25–64 is 0.58 using NTA weights and 0.29 using the microdata regression. The scale therefore serves as a reference for the cost gradient, not as expenditure: without recognition and a return on investment, large families are viable only with lower consumption or external assistance. Separating these explanations requires household budget microdata by number of children (ENIGH 2018; files unavailable).

6.6 Cost and Return on Investment in Children

Table 26 - Cost and return per child (Costa Rica, NTA 2013).

Item (Costa Rica, NTA 2013)	Value	Who bears or receives it
Private cost per child aged 0–22	41% of a Structurer's income per year	The parents
Public cost borne by Structurers	an additional 14% of a Structurer's income per year	The Structurers (taxes)
Private return: net private transfers from Ascendants	-11.0% of consumption at ages 61+ (-1.9% from age 75)	Practically nil for the parents
Public return: public transfers to Ascendants	54.4% of consumption at ages 61+; age profile, not by number of children	Collective, not individualized
Demand return (spending becomes third parties' income)	40% of the cost (ceiling)	Third parties, via labor income

Source: CCP-UCR (CNT_CR2013) and OECD (ICIO 2019).

Interpretation and limitations (return). The return on investment in children, as measured by Costa Rica's 2013 NTA, is nil for parents or collective (Table 26). Private: net private transfers from Ascendants equal −11.0% of their consumption (−1.9% from age 75 onward); that is, they do not live on transfers from their adult children. Public: 54.4% of Ascendants' consumption is financed by public transfers, and the profile is by age, not by number of children; country-specific rules (pension or tax credits per child) were not verified. Demand: spending on the child returns as income to third parties, equivalent to 40% of the cost (upper bound). In short, the cost is private and concentrated on parents, while the return is collective or absent. The article's hypothesis is that, without recognition and a return on investment to URSS, Structurers bear the entire burden, have fewer children, and this compromises generational sustainability; NTA shows the asymmetry, not the reasons for fertility decisions.

7. DISCUSSION

The comparative results reveal three distinct outcomes under the Target scenario: only Italy reaches the equilibrium zone (1.01), South Korea approaches the lower threshold (0.90), and Costa Rica leaves the equilibrium range, declining to 0.88. In none of the three countries does fertility convergence restore the 2024 indicators.

Italy. It is the only one of the three countries in which N* reaches equilibrium under the Target scenario (1.01 in 2099), after a trough of 0.59 between 2030 and 2100. With the UN TFR at 1.21 in 2024 and a projection rising above 1.3 in 2049, the C30/D70 profile is consistent with the TFR rule,

although the choice between D and E is the author's. The cutoffs are the same as in C50/D50, so the result does not depend on this choice. Italy is also the only country with a Directory value for employment per child (0.251 per year, an upper bound; 0.19 under the method applied to all three countries), making Italy the pilot case for complete accounts.

South Korea. This is the case of most extreme fertility (0.73 in 2024), and the UN projection keeps it below 1.3 through 2099. Under the Target scenario, N* reaches 0.902 in 2099, but the trough of 0.58 in 2037 is unavoidable: the adults of that period have already been born. The 0.902 result is interpreted as being at the boundary of the 0.90 threshold. The average consumption of a Descendant (US$20,800 in 2019) is 19% above the national average, and the employment intensity of consumption is high (15.1 per US$ million): the upper bound is 0.31 gross jobs per additional child per year, the highest of the three.

Costa Rica. It is the only one of the three countries that starts in equilibrium (1.32), due to a young inherited age structure resulting from a rapid demographic transition: TFR fell from 3.17 in 1990 to 1.32 in 2024 (ECLAC, 2025). Under the Target scenario, N* falls to 0.80 in 2074 and settles at 0.88; longevity (life expectancy of approximately 90 years in 2099 in the UN projection) reduces N* by construction, and replacement fertility does not bring it to equilibrium. Activity at age 65 and over is low (17.0% of men and 5.2% of women employed in July 2024; Espinoza-Herrera and Méndez-Castellanos, 2026), consistent with a retirement cutoff close to age 60. The national TFR (approximately 1.2 according to INEC) would place it in category D, while the UN figure (1.32) places it at the C/D boundary, supporting C50/D50. The national accounts data are the weakest of the three countries. The upper bound for gross employment per additional child is 0.20 per year (0.184 to 0.202 across the 2008 and 2013 CCP-UCR profiles): employment intensity of consumption is the highest of the three (25.7 per US$ million), but consumption per Descendant is approximately two-fifths of the Italian figure.

Private cost and collective benefit. In Costa Rica, NTA shows the cost structure of reproduction: Descendants' consumption is financed mainly by private transfers—that is, by parents—whereas Ascendants' consumption is financed mainly by public transfers. The cost of raising children is private, while the benefit—the future workers and taxpayers—is collective. In the language of this study, parent-Structurers forgo investment in themselves in order to invest in their children, setting the socioeconomic structure in motion, and bear the burden throughout the generation separating birth from entry into the labor force; this is the bottom of the J-curve (Sections 6.3 and 6.4). NTA shows the asymmetry of costs, not the reasons behind fertility decisions, which this article does not test.

Generational circuit. As a conceptual image, the generational circuit is a chain of three links that sustain one another cyclically: the contraction of the children's link and the extension of the Ascendants' link compress the Structurers' link, which accounts for 87% to 91% of labor income (Section 6.3). With fewer Structurers per dependent, the cost per active person rises, and the hypothesis is that this further reduces births, feeding back into the trend. The data in this article show the first two movements—the compression of the central link and the concentration of costs on parents; the feedback effect is a hypothesis, not a finding. This sense of circularity (reciprocity among cohorts) differs from the methodological circularity avoided in Section 1.

Longevity and profile adjustment. If life expectancy and activity at older ages increase and subsequently stabilize, the Ascendant cutoff and the equilibrium N* adjust. The profiles and

falsifiability test are designed to track this movement. However, the adjustment has scientific value only if it follows an observable fixed in advance (Section 4.6); otherwise, the cutoff would become a degree of freedom capable of placing any country in equilibrium. In the UN projection, Costa Rica's life expectancy reaches approximately 90 years in 2099, meaning that a possible stabilization point is not observable within the study horizon.

Return on investment and generational sustainability. The cost of raising children is private and increases with the number of children; the return is zero, collective, or not appropriable by parents (Section 6.6). If the value generated by investment in children does not reach the URSS units that make that investment, Structurers bear the entire burden, tend to have fewer children, and the base supporting Descendants and Ascendants shrinks, feeding back into the generational circuit. This is the article's central hypothesis concerning the difficulty of sustaining replacement fertility; the data show the asymmetry of costs and returns, not behavior. Designing return mechanisms (for example, pension or tax credits proportional to the number of children) is beyond the scope of this article, but Table 26 indicates where they would need to operate: in private returns and individualized public returns, currently zero in the accounts.

URSS circularity (hypothesis). Children's demand activates production chains and, with a lag, forms the future base of workers and taxpayers, but the return is distributed among third parties and the public sector and does not reach the investing unit in individualized form (Table 26). If recognition and compensation mechanisms returned part of this value to URSS units, they could reduce the private concentration of costs and encourage the next round of generational investment. This is an institutional hypothesis, not a finding: Costa Rica's data show the asymmetry, not the effect of any mechanism on fertility, net employment, or well-being. Recognizing the distinct and complementary contributions of families, without presuming who provides them, is the normative principle (contributory parity) guiding this agenda; its design is left for future work.

What the results suggest, under the stated conditions. (i) Below 2.1, the situation is critical (2099 N* under the Trend scenario ranges from 0.29 to 0.46); replacement fertility at 2.1 is a necessary condition for stabilization and avoids the most destructive trajectory, but does not restore the 2024 ratios or, by itself, guarantee N* equilibrium (only Italy reaches it in 2099). (ii) Where the inherited young age structure is large and longevity is high, as in Costa Rica, N* equilibrium requires more than replacement fertility (a target above 2.2 or additional migration of young adults). (iii) Where the adults at the trough have already been born, as in South Korea, none of the tested pathways changes the trough substantially. These are findings about the construction of the index, not evidence about employment or labor income; the link to employment is the subject of the economic estimation that remains to be completed.

The jobs that will be born. Employment generated by children is not automatic. Demand from one additional child supports 0.19 to 0.31 gross jobs per year, but this effect is completed only if the circuit closes: if replacement fertility stabilizes the base, if the burden of investing in children does not remain entirely with those making the investment, and if the return on that investment reaches the URSS units. The creation of potential jobs therefore depends on institutional closure of the circuit and redistribution of costs currently concentrated in families.

8. LIMITATIONS AND SAFEGUARDS

- N* depends on TFR, migration, mortality, and age cutoffs; its trajectory is a demographic identity, not evidence. The zones are provisional (the PQDF-N* F4 robustness test, described in Section 2.5, is not satisfied).
- The sample was selected after the demographic exploration (n = 3, descriptive). Costa Rica has the weakest national data: NTA from 2013 (the 2018 data were not obtained), and the BCCR's 2022 Supply-Use/Input-Output Tables and INEC's ENIGH 2018 are inaccessible at the file level.
- Profile composition is the author's judgment, and the TFR rule does not distinguish D from E. It is fixed over time in the engine. National TFR values (ISTAT, KOSTAT, INEC) were not read from official documents; age-specific activity data were obtained for Italy and South Korea (ILOSTAT 2025; Section 4.6), but not for Costa Rica, and N* using the new cutoff was not calculated because the proxy threshold still needs to be fixed.
- Claims concerning reductions in jobs and labor income in countries outside equilibrium are hypotheses; Section 6.3 measures only the mechanical demographic effect on Structurers' income, without productivity or participation; the cost of children is entered through the children's life-cycle deficit, borne entirely by Structurers (an upper bound), with or without the demand return, which is itself an upper bound (Table 22). Productivity gains and longevity as factors that mask the dynamics are also hypotheses.
- Employment values are demand upper bounds (with neither displacement nor labor supply) and use uniform employment intensities by section. VADc and employment use the 2019 ICIO, not more recent national matrices. Employment per child is an average, not marginal, number of employed persons (not full-time equivalents); in Italy, the method yields 26% less than the Directory; in Costa Rica, the profile is from 2013 CCP-UCR, while the 2008 profile yields a lower value (Table 19).
- The gradient of cost by number of children is a linear extrapolation of the average NTA cost and fails as an expenditure estimate for three or more children (Section 6.5); the cost allocated by NTA uses equivalence weights and may exceed actual expenditure. Private and public returns were read from Costa Rica's 2013 NTA age profiles, and country-specific rules (pension or tax credits per child) were not verified.
- Attributing exports to consumption is a rule (balanced trade), not a causal result; exports depend on the demographics of trading partners, which were not measured; investment was not allocated (Section 6.1).
- NTA shows the cost structure of reproduction (who pays), not the reasons for fertility decisions; only Costa Rica has a digital profile of private and public transfers, and the measured burden does not include care time (time-use accounts).
- The demographic exploration uses a single scenario (a 25-year ramp-up, target 2.1), with UN migration and mortality assumptions and no uncertainty interval. Period TFR may be affected by tempo effects.
- The reference NTA datasets are from different years (Italy 2008; South Korea 2024, from KOSIS, with 2000 savings and 2010 weights in the Directory; Costa Rica 2013), limiting cross-country comparison. CFCc uses smoothed profiles (Italy) or unsmoothed profiles (South Korea) and UN population data, and extends the final age of the profile to older ages.

- The estimate of approximately 1.5 jobs per couple with two children is an order-of-magnitude proposition by the author, not a result from the tables.
- Care time and unpaid production are excluded from the monetary accounts used; their relevance is drawn from the cited literature (Jiménez-Fontana, 2015), without re-estimation in this article. The recorded private burden is therefore a lower bound on the actual cost.
- NTA profiles are averages by age and do not separate those who have children from those who do not; for South Korea, Hwang and Yoo (2025) suggest that mothers' earnings losses and selection into motherhood have changed across cohorts, so the J-curve estimated here is an average profile, not the causal effect of having children.

9. FINAL CONSIDERATIONS

The study begins with an accounting observation: each additional birth introduces a new consumer into the economy. According to the preliminary calculations, this demand supports approximately 0.2 to 0.3 gross jobs per child per year. A couple with two children generates, through the children's demand alone, about half a job and, when the parents' continued presence in the Structurers cohort is included, the complete cycle approaches 1.5 jobs in order of magnitude. These are the jobs that will be born, provided that the circuit closes.

The demographic analysis shows three trajectories under the same fertility target. Italy reaches N* equilibrium in 2099 (1.01). South Korea reaches the threshold boundary (0.902), with a trough of 0.58 in 2037 that none of the tested pathways eliminates. Costa Rica, which starts in equilibrium (1.32), leaves it and settles at 0.88. In none of the three countries does the Target restore the 2024 ratios: the 2099 ratio of Ascendants to Structurers is 0.72 to 0.78, above the 2024 ratio, and potential caregivers per person aged 80 or over fall by 51% to 86%.

Five conclusions hold under the stated caveats:

- Demographic equilibrium is a process that takes decades: the Target avoids the most destructive trajectory, but the cost of the transition is already embedded in the cohorts that have been born.
- Replacement TFR is a basic requirement, but it does not by itself guarantee system stability: where the inherited age structure is young and longevity is high, as in Costa Rica, a target of approximately 2.2 to 2.35 or additional migration of young adults would be necessary.
- Judgments about structural profiles move N* by hundredths, and the compositions, rules, and evidence are documented and open to challenge.
- The burden of the transition falls primarily on parents: the cost of additional children is largely private, and the J-curve of Structurers' income has a bottom that lasts more than four decades before the Target surpasses the Trend.
- The return on investment in children is zero for parents or collective, and the extrapolated cost gradient by number of children is inconsistent with actual expenditure. Without recognition and returns for URSS units, the burden falls on Structurers, and this is the central hypothesis concerning the difficulty of sustaining replacement fertility.

The article does not claim that replacement fertility is sufficient or that cost is the reason for low fertility. It argues that below 2.1 the demographic picture is critical (2099 N* under the Trend scenario ranges from 0.29 to 0.46), that 2.1 is the minimum condition for stabilization, and that the cost of transition is measured through transfers and deficits recorded by NTA.

Costa Rica's case summarizes this impasse. In addition to the private financing recorded by NTA (61.4% of Descendants' consumption), unpaid work and inequality in care, identified by Jiménez-Fontana (2015) and Filgueira and Martínez Franzoni (2019), increase the actual cost borne by families. Therefore, stabilizing demographic indicators is not enough: future jobs will materialize sustainably only if investment in educating new generations is no longer treated as an almost exclusively private burden.

What this article does not show. It does not yet show that a country's position in N* explains employment, labor income, or well-being: the net-employment table remains to be completed; Structurers' income (Sections 6.3 and 6.4) is demographic accounting with the cost of children based on NTA, without productivity or participation; CFCc is populated for all three countries (Costa Rica using 2013 NTA); and the upper bound for gross jobs per additional child (0.19, 0.31, and 0.20 per year in Italy, South Korea, and Costa Rica) is preliminary: average consumption, with no displacement and no estimate of net employment. It does not show a statistical relationship between N* and outcomes, because n = 3 and the countries were selected after the exploration. Nor is it a forecast: the scenarios are conditional and use a single set of assumptions.

Next steps. (i) Obtain Costa Rica's 2018 NTA, which was not located in numerical form, and verify the 2013 profile with CCP-UCR, currently the country's basis in Table 16. (ii) Re-estimate VADc and EDPc using more recent national matrices (BCCR's 2022 Input-Output/Supply-Use Tables) and cohort allocation rules. (iii) Calculate net employment under 0%, 50%, and 100% displacement using marginal consumption (Table 19 presents the upper bound for all three countries, 50% displacement as a proportional fraction, and 100% only for Italy, based on the Directory). (iv) Measure private and public transfers for children in Italy and South Korea (currently only the life-cycle deficit is available), include care time (time-use accounts), productivity and participation, and measure the return from demand under conditions of spare capacity; use household budget microdata (ENIGH 2018 and SHIW) to distinguish the three explanations for the implausible cost of large families (lower consumption per child, assistance from third parties, higher income); and identify mechanisms for returns to parents (pension or tax credits per child) in each country's rules. (v) Read national TFR data (ISTAT, KOSTAT, INEC); set and preregister the activity-proxy threshold (Section 4.6) before calculating N* with the new cutoff; obtain Costa Rica's age-specific activity ranges; and weight exports by the demographics of trading partners. (vi) Decide whether to reformulate H8 and set and preregister the thresholds for H1, H3, H5, and H8 before economic estimation. (vii) Consider time-varying profiles in the engine. (viii) Address the design and modeling of compensation mechanisms, with contributory parity as the guiding principle, in a separate article.

AUTHORSHIP AND AI USE STATEMENT

The ideas, structure of the paper, writing style, theoretical foundation, Narayama Systemic Index (N*), formulas, sample definition, and interpretation of results are the author's work. Artificial intelligence tools were used for translation, image generation, and language editing, under the author's review. The calculations are based on the sources listed under "Data Sources"; the demographic engine and N*_Obs data are available in the public AINU.Systems repository (see "Data and Code Availability").

DATA AND CODE AVAILABILITY

The demographic engine, processed N*_Obs data, and profile composition are available in the public AINU.Systems repository (https://github.com/itibere-muarrek/Ainu-Narayama), version v0.1.0 (Git tag, November 14, 2026), and on ainu.systems and narayama.live. The code is licensed under the MIT License, and derived data and documentation under CC BY 4.0; third-party data remain subject to the terms of each source (files LICENSE, LICENSE-DATA.md, and THIRD_PARTY_DATA.md). The routines for the economic calculations in this article (employment per child using ICIO, consumption by cohort, Structurers' labor income, cost of children, and employment decomposition) are not published in this version; the methods are described in Sections 4 to 6, and the results can be checked using the sources listed under "Data Sources." The raw ICIO, NTA, and CCP-UCR files are not redistributed with the article.

ACRONYMS AND ABBREVIATIONS

BCCR, Central Bank of Costa Rica; CCP-UCR, Central American Population Center, University of Costa Rica; ECLAC, Economic Commission for Latin America and the Caribbean; CFCc, Cohort Final Consumption; ENAHO, National Household Survey (Costa Rica); ENIGH, National Household Income and Expenditure Survey (Costa Rica); ICIO, Inter-Country Input-Output tables, OECD international input-output tables; ILOSTAT, ILO statistical database; INEC, National Institute of Statistics and Censuses (Costa Rica); ISIC, International Standard Industrial Classification, Revision 4; ISTAT, Italian National Institute of Statistics; KOSTAT, Statistics Korea; MIP/COU, input-output matrix and supply-use tables; NTA, National Transfer Accounts; OECD, Organisation for Economic Co-operation and Development; ILO, International Labour Organization; UN, United Nations; PEA, Self-Sustaining Equilibrium Point (PEA); PEC, Narayama Collapse (PEC); PQDF-N*, N* Quality, Decomposition, and Falsifiability Protocol; SHIW, Survey on Household Income and Wealth (Banca d'Italia); TFR, total fertility rate; URSS, Sustainable Social Reproduction Unit (original Portuguese acronym); WPP, World Population Prospects.

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Data Sources

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employment intensity of final consumption, and employment decomposition by demand component (Sections 6.1 and 6.2).

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AINU.Systems. Cohort projection engine, profile composition (C30/D70 for Italy and South Korea; C50/D50 for Costa Rica), and data and simulation directory. https://ainu.systems and https://narayama.live, accessed November 11, 2026. Use: N*_Obs, Trend and Target trajectories, A/E and E/80+ ratios, and the Directory value for Italy (Sections 4 to 6).

Note. National fertility rates (INEC, ISTAT, KOSTAT) and activity at ages 60–64 (ENAHO, ISTAT, KOSTAT) were not read from official documents (L2 level) and therefore do not support conclusions; fertility values come from UN WPP 2024. Calculations of employment per child, Structurers' labor income, and the cost of children follow the methods described in Sections 4 to 6, based on the files listed above; the routines for these calculations are not published in this version.