

Glocalization Destroyed, Generational Reproduction Collapsed: The Forest as a Demographic Tombstone in Small and Medium Italian Cities

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ABSTRACT

This paper applies the Narayama Systemic framework to the Italian case to demonstrate that the spontaneous advance of forests over historically cultivated land does not constitute an environmental phenomenon, but the geographic signature of the collapse of the Sustainable Social Reproductive Unit (URSS) in small and medium-sized municipalities. We argue that the destruction of Glocalization (Factor L) — driven by urban economic concentration, interior deindustrialization, and the systematic dismantling of local public services — is the root cause of Italy's demographic collapse ($TFR = 1.14$; $N^* = 0.49$; PEC Zone). The mechanism unfolds in three stages: (1) L falls → URSS loses economic viability; (2) Generational Provider migrates → municipalities empty; (3) forest advances → cultural landscape collapses. We propose that small and medium cities with high L simultaneously resolve all five pillars of the Narayama Pentagon, and that Italian forests — now covering 40% of the national territory — are the geographic photograph of URSSs that never formed. The IVAT (Technological Value-Added Tax) and active Glocalization are presented as the two structuralbiological mechanisms capable of reversing this trajectory before the reversibility window closes.

Keywords: Narayama Systemic; Glocalization; URSS; Italy; Borghi Abbandonati; TFR; IVAT; Demographic Collapse; Small Cities.

1. INTRODUCTION: THE ACCUSING FOREST PARADOX

In 2020, for the first time since the Middle Ages, Italy's forest cover officially exceeded the agricultural area in use. Today, 11.4 million hectares — 38% to 40% of the national territory — are covered by woodland (INFC, 2023). Seventy-five percent of this forest is concentrated in mountainous municipalities where only 1% of the Italian population lives (ISTAT, 2024). This phenomenon is systematically celebrated as an environmental achievement.

This paper argues the opposite: the Italian forest is a demographic tombstone. It did not grow because Italy chose nature — it grew because Italy lost the people who cultivated the land, raised

children, and sustained communities. Every hectare of spontaneous forest is the geographic trace of a Sustainable Social Reproductive Unit (URSS) that never existed.

Italy today records a Total Fertility Rate (TFR) of 1.14 — among the lowest in the developed world (ISTAT, 2025). According to the Narayama Systemic Index (N^*), calculated in this paper at 0.49, the country is in the PEC Zone (Critical Entropy Point): structural demographic collapse with endogenous reversal of difficult execution. The question guiding this work is not "why don't Italians have children?" — but "why were the structural conditions for having children destroyed, and where is that destruction most visible?"

The answer lies in the small and medium cities of the Italian interior — the borghi abbandonati, the Apennine and Alpine communities emptied by rural exodus, deindustrialization, and the systematic dismantling of local public services. In these territories, the Glocalization Factor (L) collapsed — and with it, the economic viability of the URSS, community cohesion, the Natural Health Index (NIH), and the intergenerational transmission of knowledge and identity.

This paper is organized into six sections. Section 2 presents the Narayama Systemic framework and its operational instruments. Section 3 diagnoses the Italian case with empirical data. Section 4 develops the central hypothesis: small and medium cities with high L as the architecture that simultaneously resolves all five pillars of the Pentagon. Section 5 evaluates the insufficiency of conventional policies — in particular Smart Working and the Assegno Unico. Section 6 proposes IVAT and active Glocalization as structural reversal mechanisms.

2. ANALYTICAL FRAMEWORK: THE NARAYAMA SYSTEMIC AND FACTOR L

2.1 The Narayama Systemic Index (N^*)

The Narayama Systemic is an integrated framework for diagnosing intergenerational sustainability developed by Muarrek (2026). Its central instrument is the Narayama Systemic Index (N^*), defined as:

$$N^* = \sqrt{(NGII_puro \times Fator_Geracional)}$$

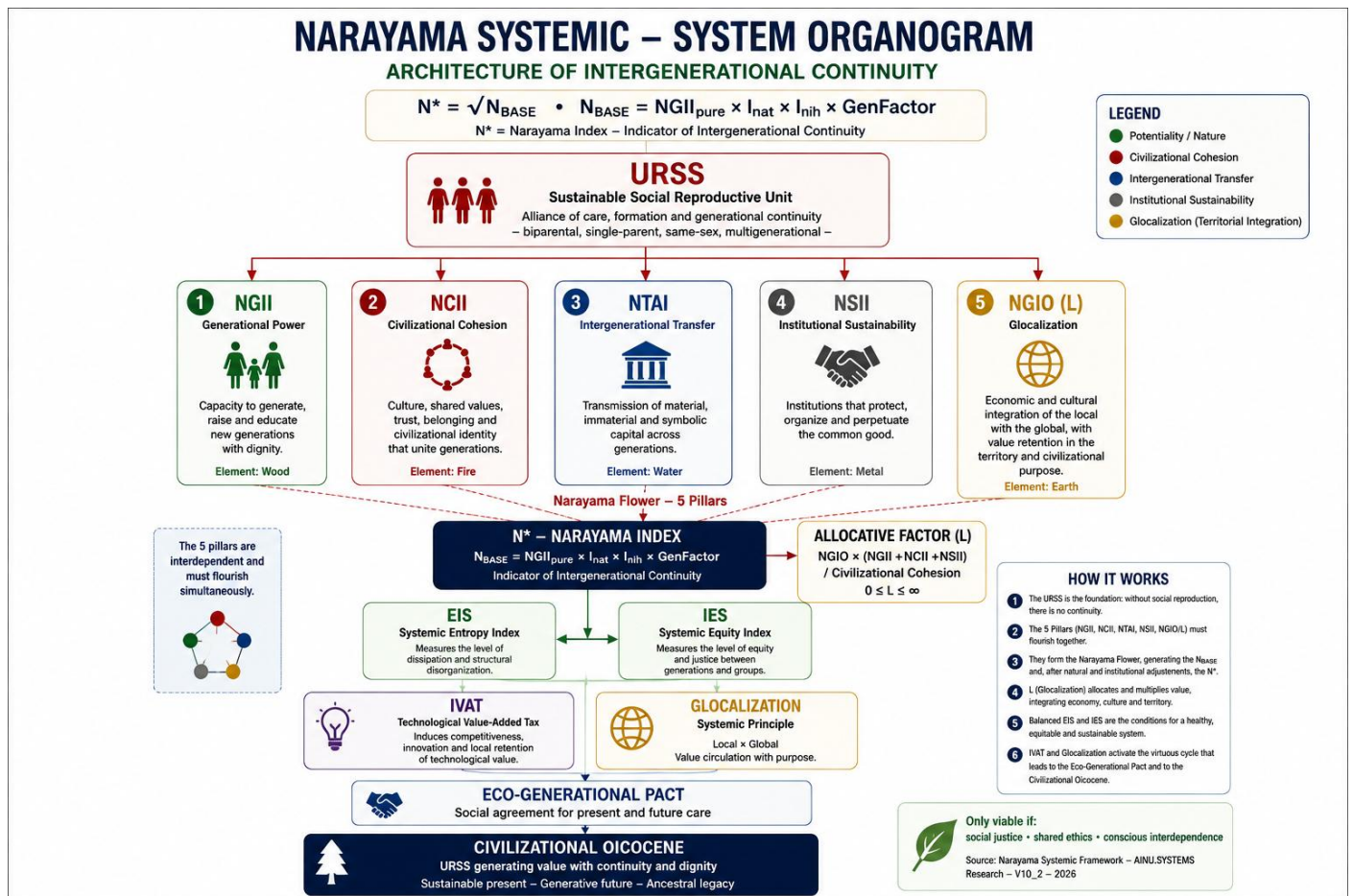
where NGII_puro measures endogenous reproductive strength (ratio between the formative cohort and the dependent cohort, weighted by the births/deaths ratio) and Fator_Geracional captures the intergenerational trajectory of fertility (TFR_t / TFR_{t-25}). N^* positions each society in one of five generational regimes: PEEC (saturation), Population Tension, PEA (sustainable equilibrium), Accelerated Tension, and PEC (collapse).

For Italy (2024):

- NGII_puro = 0.2489;
- Fator_Geracional = 0.982;
- N_Base = 0.244;
- **$N^* = 0.49$ — PEC Zone.**

This result places Italy among the most critical cases in the comparative sample of 28 countries, alongside Japan ($N^* = 0.43$) and South Korea ($N^* = 0.37$). The calculations and comparative base are part of the Narayama research (Muarrek, 2026).

Figure 1 presents the functional architecture of the system and the position of N* in relation to the other framework components.



2.2 The Sustainable Social Reproductive Unit (URSS)

The Sustainable Social Reproductive Unit (URSS) is the basic unit of social reproduction and generational continuity, whose biological foundation is procreation between man and woman — male and female — the condition of human sexual reproduction. Upon this nucleus, different social and family configurations of care, upbringing, formation, and intergenerational transmission may adhere and organize themselves, including configurations that, by their very constitution, do not possess endogenous reproductive capacity.

These configurations may participate in raising, caring for, and forming new generations, but cannot, in isolation, biologically generate a new generation. Their population continuity depends on the incorporation of children, individuals, or reproductive material originating outside the configuration itself — through adoption, immigration, or assisted reproduction with external gametes.

The distinction is biological, not moral, cultural, or ideological: different forms may perform social functions of reproduction and transmission, but the endogenous biological renewal of the human population remains dependent on sexual reproduction between the two reproductive sexes.

It is precisely this difference between social reproduction ≠ biological reproduction that makes the Narayama Dilemma possible: a society may continue caring for and transmitting its culture while progressively losing its capacity to endogenously produce the next generation.

2.3 Factor L (Glocalization) and the NIH

Factor L operationalizes the fifth pillar of the Narayama Pentagon — the NGIO (Glocalization and Sovereignty). It measures the local capacity for economic value retention, food autonomy, and territorial resilience:

$$L(i,t) = f(\text{RET}, \text{COMP}, \text{SYM})$$

where RET is local value retention ($1 - \text{imports}/\text{local GDP}$), COMP is local economic complexity and SYM the income distribution symmetry within the territory. L acts transversally in the IES formula (Systemic Equilibrium Index), amplifying or reducing the combined effect of the remaining pillars.

The Natural Health Index (NIH) is a bioeconomic component of Factor L. It measures the biological cost imposed on the Provider by the prevailing productive and dietary model: exposure to ultra-processed foods, endocrine disruptors, chronic medicalization, and systemic inflammation. Small and medium cities with preserved local food production present structurally higher NIH — and, consequently, higher effective fertility and lower URSS formation cost.

Factor L, therefore, does not constitute merely a territorial variable: it represents the capacity of a community to locally maintain the material conditions that sustain social reproduction.

2.4 The Pentagon and the Integrated Causal Mechanism

It is at this point that the components cease to operate as isolated variables and begin to form the Narayama system.

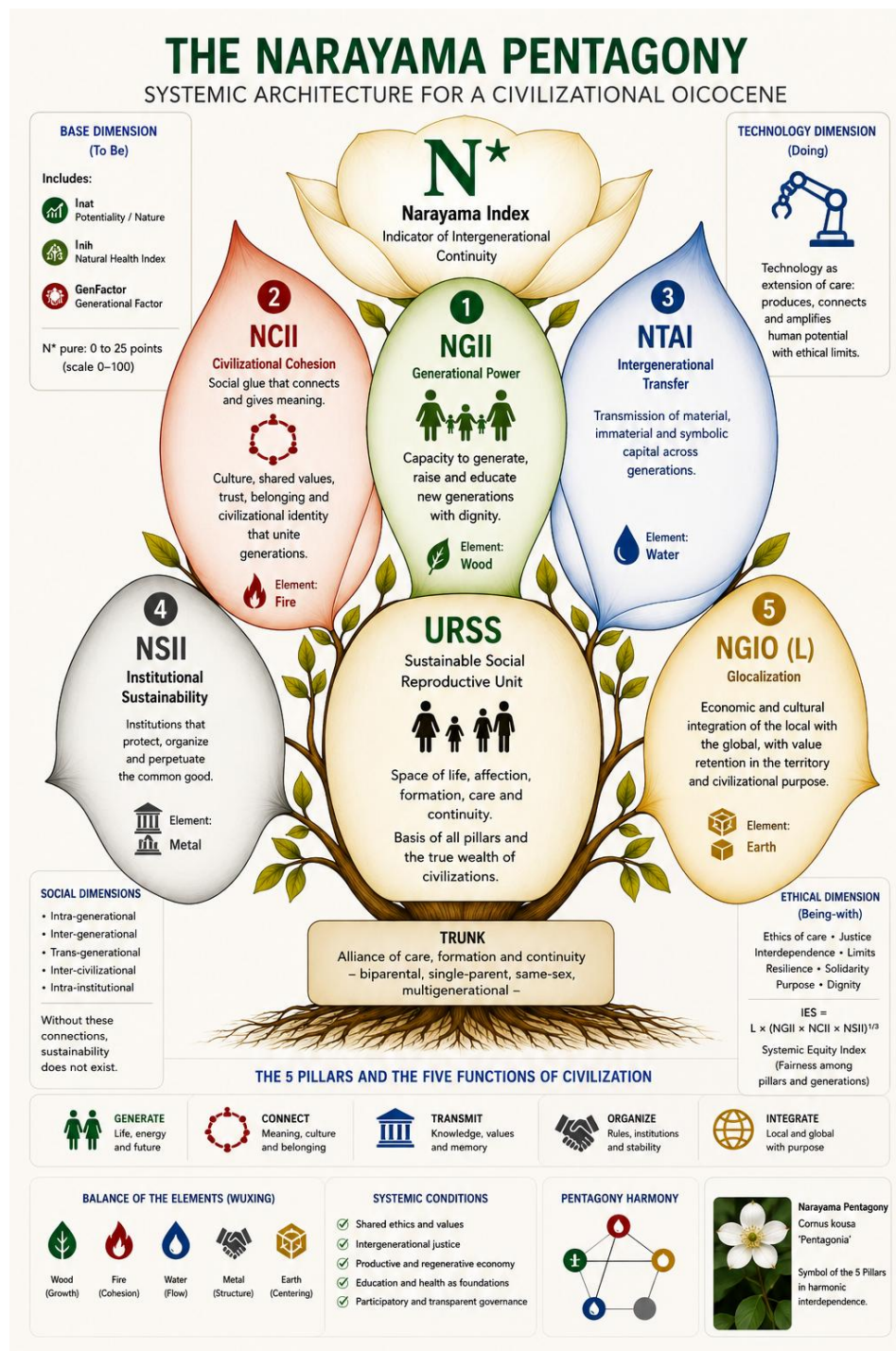
The framework proposes that intergenerational sustainability emerges from the simultaneous flourishing of five dimensions — the Narayama Pentagon: Generational Potency (NGII), Civilizational Cohesion (NCII), Intergenerational Transfer (NTAI), Institutional Sustainability (NSII), and Glocalization (NGIO/L).

The central causal mechanism of this paper can be formalized as follows:

- L falls → Provider's cost of living rises + decent work becomes scarce
- URSS loses viability → reproductive decision becomes rationally unviable
- NGII falls → N^* deteriorates → trajectory toward PEC
- NCII falls → community cohesion dissolves → migration accelerates
- Municipality empties → forest advances → cultural landscape collapses
- Gerontocracy without support → structural pension insolvency

This mechanism is neither linear nor monocausal — it is systemic and self-reinforcing. Once initiated, each stage of deterioration accelerates the others. The reversibility window is measured in years, not decades.

Figure 2 synthesizes the expanded systemic architecture, in which N^* , the URSS and the remaining domains are articulated around the sustainable intergenerational flow.



3. THE ITALIAN CASE: DATA, N* AND THE FOREST AS AN INVERSE INDICATOR OF L

3.1 Demographic Profile and the Italian N*

Italy (mixed age composition) presents the following consolidated indicators for 2024–2025 (ISTAT, 2024; UN WPP, 2024; AINU pipeline v3.1.0):

- TFR: 1.14 (2025) — historical floor
- N^* : 0.49 — PEC Zone (Critical Entropy Point)
- NGII_puro: 0.2489
- Fator_Geracional: 0.982 (TFR 2024 / TFR 1999)
- Median age: 48.9 years (second highest in Europe)
- Annual births: ~370,000 (2024) — historical minimum
- Pension expenditure: ~17.2% of GDP — highest in the EU
- Estimated Fator_Alocativo: < 0.50 — Farol (–) — predominantly gerontocratic
- Projected dependency ratio (2050): 68.5% (Eurostat)

With $N^* = 0.49$, Italy stands well below the Accelerated Tension threshold (0.71) and approaches the Japanese level (0.43). The AINU projection (Scenario A — inaction) points to $N^* = 0.38$ in 2035 and a loss of 4 million inhabitants by 2050 (ISTAT, 2025).

3.2 The Forest as an Inverse Indicator of Factor L

The INFC (Inventario Nazionale delle Foreste e dei Serbatoi di Carbonio) documents that Italian forest cover has doubled since 1950: from 5.6 to 11.4 million hectares. In 2020, for the first time since the Middle Ages, forest area exceeded agricultural land in use. The growth did not result from reforestation policy — it was spontaneous renaturalization due to human abandonment (CREA, 2023; MASAF, 2024).

The geographic distribution is revealing: 75.7% of the forest is located in mountainous municipalities where only 1% of the population lives. Over the last two decades, 850,000 hectares of cultivated land were lost exclusively to abandonment — not to urbanization (IUTI, 2024).

This paper proposes that spontaneous forest cover functions as an inverse proxy of Factor L at the municipal level: the further forest advances without human management, the lower the effective L of that region. The proposition is verifiable with ISTAT data by municipality — and constitutes the central empirical hypothesis of this work:

H1: Italian municipalities with $L > 0.65$ (preserved local economy, services present, active family farming) systematically present TFR closer to 2.1 and lower spontaneous forest cover, compared to municipalities with $L < 0.40$.

3.3 The Fator_Alocativo and the Suicidal Gerontocracy

Italy allocates ~17.2% of GDP to pensions — the highest proportion in the European Union. At the same time, investment in pre-school education and family support represents a marginal fraction of the budget. The estimated Fator_Alocativo ($NTA(0-25) / NTA(65+)$) is well below 0.50 — Farol (–), an indicator of predominantly allocative gerontocracy.

The paradox that this framework calls suicidal gerontocracy manifests with precision in the Italian case: the elderly cohort "won" the allocative battle over decades — securing generous pensions and robust institutional protection. But in doing so, it eliminated the Generational Providers who would sustain it.

The allocative victory of Italian gerontocracy is demographically autophagic. In 2050, with a dependency ratio of 68.5%, there will be fewer than 1.5 active workers per pensioner — and the pay-as-you-go system will face structural insolvency (Eurostat, 2024).

3.4 North vs. South: Heterogeneity as a Natural Test

Italy offers an internal natural experiment: the structural difference between the industrialized North (Emilia-Romagna, Veneto, Lombardia) and the Mezzogiorno (Calabria, Campania, Sicilia). The North preserved, for longer, networks of small and medium enterprises with territorial rooting (the *distretti industriali*). The South was emptied earlier — and today presents TFR below the national average, higher spontaneous forest cover, and a higher projected dependency ratio.

This internal heterogeneity allows for a quasi-experiment: Northern municipalities with preserved *distretti* vs. Southern municipalities with collapsed economic bases. The hypothesis is that the Factor L of the former is systematically higher, and that this is reflected in higher TFR and lesser forest advance.

4. SMALL AND MEDIUM CITIES: THE ARCHITECTURE THAT RESOLVES ALL FIVE PILLARS

This paper proposes that small and medium cities with high Factor L constitute the natural urban architecture that simultaneously resolves all five pillars of the Narayama Pentagony — without requiring intervention on each pillar in isolation. The mechanism is structural, not cultural.

4.1 Pillar 1 (NGII): Affordable m² and space for children

Housing cost is one of the most robust structural determinants of the reproductive decision (Muarrek, 2026; Eurostat, 2023). In small and medium Italian cities, the cost per m² is systematically 60–80% lower than in Milan or Rome. This has direct and measurable consequences:

- The URSS is physically viable: there is space for children, a yard, intergenerational cohabitation.
- Housing cost does not compete with the reproductive decision: one more room is affordable.
- The Provider does not need two maximum salaries to sustain a family — a local income is sufficient.

The derived hypothesis (H2) is that municipalities with an average housing cost below €1,500/m² present significantly higher TFR than municipalities above €3,000/m², controlling for income and education.

4.2 Pillar 2 (NCII): Community security and civic knowledge

Civilizational cohesion (NCII) is not produced by public policy — it is an emergent product of communities where the social fabric is dense, ties are multigenerational, and civic knowledge (who is who, where they are, how things work) is shared. In small Italian cities, this fabric still exists where emptying has not been total.

Community security reduces the cost of CHR formation: the child circulates, is supervised by the surroundings, learns through civic participation. The local capitale sociale — in Putnam's (2000) sense — acts as a multiplier of parental investment. One hour of parental care in a cohesive community produces more CHR than the same hour in urban isolation.

4.3 Pillar 3 (NTAI): The active elder as agent — not dependent

This is the most counterintuitive and most powerful point of the argument. In small cities, the Legatário (65+) is not a passive repository of pension transfers — they are an active community agent: they care for grandchildren, transmit artisanal and agricultural knowledge, maintain local institutional memory, participate in civic life.

This role liberates the Generational Provider. When the grandmother cares for the grandchild, the mother can work without paying for childcare. When the grandfather transmits the craft, the young person does not need to pay for training. The Fator_Alocativo balances organically — without public policy, without fiscal transfers, without bureaucracy. The intergenerational flow functions because physical proximity makes it possible.

In large cities, this proximity disappears. The elder goes to the nursing home (casa di riposo), the grandchild goes to paid daycare, the Provider works 60 hours a week to finance both. The Fator_Alocativo collapses — and systemic burnout sets in.

4.4 Pillar 4 (NSII): Functioning local institutions

Institutional sustainability (NSII) in small cities is more fragile in terms of absolute resources, but more effective in terms of proximity to the citizen. A local school prevents the family from emigrating. A functioning health post ensures the Provider does not lose workdays taking a child to another city. A local bank branch allows credit for small producers.

The dismantling of these services — under the logic of fiscal consolidation that closed first-cycle schools, medical posts, and bank branches in Italian borghi — was the final trigger for emptying. It was not a lack of willingness among young people to stay: it was the elimination of the minimum conditions for staying.

4.5 Pillar 5 (NGIO/L): Healthy life, high NIH and local production

Small cities with active local economies present structurally higher NIH: access to fresh food, lower exposure to ultra-processed products, lower chronic stress, lower burnout rate. These factors have direct and measurable impact on effective fertility — not just on declared intention to have children.

Outdoor life, a less compulsive pace, physical space, and contact with managed (not abandoned) nature produce a Provider health profile that reduces the biological cost of reproduction. High NIH is not rural romanticism — it is an economic mechanism: lower medicalization, lower absenteeism, lower public health costs, greater energy available for URSS formation.

4.6 The Synthesis: Why the Small City Resolves Everything Simultaneously

The table below synthesizes how small and medium cities with high L simultaneously address all five pillars, without each pillar needing to be the object of isolated public policy:

NGII: Affordable m² → URSS physically viable → TFR rises

NCII: Community cohesion → CHR produced by the surroundings → upbringing cost falls

NTAI: Active elder cares for grandchild → frees Provider → Fator_Alocativo balances organically

NSII: Local school + health post + bank → Provider does not need to emigrate

NGIO: High NIH + local production → effective fertility preserved + cost of living controlled

Central proposition (P1): The small and medium city with high L is not a public policy solution — it is an emergent architecture that organically resolves the Narayama Dilemma. The destruction of this architecture by the process of urban concentration and globalization is, therefore, the structural — not proximal — cause of Italian demographic collapse.

5. THE INSUFFICIENCY OF CONVENTIONAL POLICIES

5.1 The Assegno Unico: Transfer without Architecture

The Assegno Unico Universale, introduced in 2022, consolidated and expanded direct financial incentives for parenthood. It is the Italian version of the conventional argument: direct transfer to the Provider reduces the financial cost of having children → TFR rises.

The observed result contradicts the hypothesis: TFR fell from 1.24 (2022) to 1.14 (2025) during the period of full implementation of the benefit. The mechanism fails because it attacks the symptom (one-off financial cost) without correcting the structural cause (destruction of L, impossibility of housing, absence of local services, degraded NIH, collapsed NCII).

In Narayama Systemic language: the Assegno Unico is Pillar 2 (NTAI) operating in isolation, without Pillars 1 (NGII/housing), 3 (cohesion), 4 (NSII/services), and 5 (L/NIH). Hypothesis H4 of the framework — "the absence or fragility of any pillar creates vulnerability that the others cannot compensate" — is empirically confirmed in the Italian case.

5.2 Smart Working: Temporary Mitigator, Not Structural Reversal

Remote work was widely advocated as a vector for repopulating the Aree Interne. The Smart Working Observatory at Politecnico di Milano recorded ~3.57 million remote workers in Italy. The flow to small municipalities, however, did not follow governmental projections.

Three structural reasons explain the failure: (1) connectivity bottleneck — PNRR fiber optics arrived late and with partial coverage; (2) the illusion of work in a void — young couples who migrate to the interior abandon it within less than 24 months if they find no functioning nursery, health post, or school; (3) digital nomadism vs. demographic fixation — digital nomads inject temporary capital but do not register children, do not establish permanent residence, and do not reconstitute the community fabric.

The diagnosis is precise: Smart Working resolves the income and connectivity problem — but leaves intact the NSII (services), NCII (cohesion), and NGIO (L/NIH) problems. It is a necessary but not sufficient condition. Without the complete architecture of the five pillars, it is medium-term tourism disguised as demographic policy.

5.3 The Italian Gerontocracy Trap

The Italian case illustrates with precision the mechanism identified by Muarrek (2026) as the gerontocracy trap: in democracies with aging populations, the electoral majority belongs to older cohorts, who vote more consistently and have a direct incentive to preserve their benefits. Reforms that rebalance the Fator_Alocativo in favor of the formative cohort are electorally costly.

The tragic irony is that Italian gerontocracy is only sustainable if it is not gerontocracy. The allocative victory of the elderly cohort — securing 17.2% of GDP in pensions while services for young families were dismantled — eliminated the Providers who would sustain it. It is demographic suicide by short-term optimization.

6. IVAT AND ACTIVE GLOCALIZATION: STRUCTURAL REVERSAL MECHANISMS

6.1 IVAT as Correction of the Structural Equation

The IVAT (Technological Value-Added Tax) is the central structural mechanism proposed by the Narayama Systemic to correct the systemic isonomy rupture produced by automation and globalization. It is not a protectionist tariff — it is a self-regulating cybernetic feedback mechanism.

Its operational formula is:

$$IVAT(A/B,X) = (IES_A,X / IES_B,X) - 1$$

For the Italian case: sectors with IES below the European average receive protection via IVAT; high-technology sectors with IES above receive exemption. Revenue is directed exclusively to the Generational Provider: 40% housing for families with children, 30% direct child bonus, 20% tax compensation, 10% worker transition.

The crucial difference from the Assegno Unico: IVAT is not a transfer — it is a structural rebalancing of the cost equation. It does not compensate the Provider for the cost of having children in a hostile system: it alters the system so that having children ceases to be economically irrational.

6.2 Active Glocalization: Rebuilding L in the Borghi

Active Glocalization is not a nostalgic return to the agricultural past — it is the deliberate reconstruction of Factor L in the territories where it was destroyed. For the borghi and Italian Aree Interne, this implies:

- Reactivation of local public services (school, health post, bank branch) as a condition of habitability — not a luxury.
- Incentives for family farming and proximity agri-food distretti.
- Tariff protection via IVAT for local products against dumping from long chains.
- Preferential credit for URSSs that settle in municipalities with L below the critical threshold.
- Integration of Smart Working with complete social infrastructure — not just fiber optics.

Active Glocalization transforms spontaneous forest into managed forest — and, more importantly, creates the conditions for Providers to return to occupying the territory before the reversibility window closes definitively.

6.3 The Italian Reversibility Window

With $N^* = 0.49$, Italy is in the PEC Zone — but still above the Japanese level (0.43), which this framework considers of structurally very compromised reversibility. The Italian window exists, but is narrow and closes quickly: with the current reproductive cohort (women aged 20–40) still demographically significant, the simultaneous implementation of the three pillars (IVAT + active Glocalization + reactivation of local services) could, according to AINU simulations (Scenario C), raise N^* to 0.65–0.70 by 2035 and stabilize TFR around 1.60–1.75 — insufficient for PEA, but sufficient to interrupt the trajectory of irreversible collapse.

7. CONCLUSION: THE FLOWERING THAT LIES

The forest flourished — and it is precisely there that the deception lies. Its advance may be celebrated as environmental recovery, but in the territories analyzed by this paper, it may also represent the disappearance of the people, productive activities, and intergenerational relations that historically kept that landscape alive. What appears to be a flourishing of nature may simultaneously be the flourishing of a tombstone.

This paper argued that this phenomenon is not environmental — it is demographic, economic, and institutional. The forest is the inverse indicator of Factor L: where L collapsed, the cultural landscape disappeared and spontaneous vegetation advanced. Where L was maintained — in the northern distretti, in communities with preserved services — the territory remained alive, productive, and demographically sustainable.

The Narayama Systemic offers the operational diagnosis: $N^* = 0.49$, PEC Zone, Fator_Alocativo (–), NIH in decline, NCII fragile, L destroyed in the Aree Interne. Italian gerontocracy won the allocative battle and lost the demographic war.

The solution does not lie in more financial transfers to families — the Assegno Unico demonstrated this empirically. It lies in the reconstruction of L: in the reactivation of local services, in active Glocalization, in IVAT that rebalances competition between human labor and automation, and in the understanding that small and medium cities with a living local economy are the architecture that organically resolves the five pillars of the Narayama Pentagon.

The flowering that lies... Every hectare that advances without human management is evidence of a URSS that is gone, by the absence of a Provider, of a generation that was never born. The challenge is not to cut down the forest — it is to build the conditions for people to want to return to inhabit it.

APPENDIX A — TECHNICAL JUSTIFICATIONS FOR THE N^* CONSTRUCTION

A.1. Construction Principle

The Narayama Systemic Index (N^*) was conceived to integrate two dimensions of population renewal: the structural demographic capacity and its generational trajectory.

Its canonical formulation is:

$$N_Base(i,t) = NGII_puro(i,t) \times Fator_Geracional(i,t) \quad N^*(i,t) = \sqrt{N_Base(i,t)}$$

NGII_puro represents the structural dimension of the reproductive stock; Fator_Geracional, its evolution over time.

The objective is to prevent the diagnosis from depending exclusively on a demographic snapshot or a current flow.

A.2. Multiplicative Form and Temporal Horizon

The multiplicative form derives from the hypothesis that stock and trajectory are complementary dimensions, and not fully substitutable. An additive formulation would allow one component to fully compensate for the deterioration of the other. Multiplication reduces this possibility: the deterioration of any component affects the resulting nucleus.

The generational factor is defined by:

$$Fator_Geracional(i,t) = TFR(i,t) / TFR(i,t-25)$$

The 25-year horizon constitutes an operational approximation of the generational cycle, allowing demographic inertia to be incorporated into the indicator. It does not represent a universal biological constant, but a model time window. Its adequacy must be confronted with alternative horizons.

A.3. NGII_puro and cohorts

NGII_puro is defined as:

$$NGII_puro(i,t) = [Pop_Base / Pop_Topo] \times [Births / Deaths]$$

The first ratio represents the relationship between the population at the reproductive base of the system and that at an advanced stage of the generational cycle; the second incorporates the current dynamics between births and deaths.

The indicator does not equate to a conventional dependency ratio or a direct measure of reproductive capacity. It is a specific operational construction of N^* .

Age groups are kept constant to ensure temporal and cross-sectional comparability. Their influence on results remains subject to sensitivity analysis.

A.4. The Square Root Transformation

The transformation $N^* = \sqrt{N_Base}$ was adopted for three reasons. First, it preserves the reference point: $N_Base = 1 \Rightarrow N^* = 1$. Second, it reduces the dispersion produced by the multiplicative combination, without altering the ordering of positive values. Third, it maintains a scale of direct interpretation around the unit value.

The square root constitutes a model specification, not a mathematical necessity. Its influence will be confronted with alternative transformations in sensitivity analyses.

A.5. Separation between N^* , Farol, and Pentagony

N* was deliberately maintained as the demographic nucleus. The Farol Institucional represents a distinct dimension, related to the direction of intergenerational transfers:

$$\text{Fator_Alocativo} = \text{NTA}(0-25) / \text{NTA}(65+)$$

Keeping it separate allows distinguishing the demographic condition and trajectory from the institutional direction of transfers and testing whether this adds information to N*.

Similarly, the five pillars of the Pentagony are not directly aggregated into the formula. The distinction is:

N* = demographic nucleus

Pentagony = expanded systemic architecture

N* constitutes the central diagnostic instrument; the other components represent explanatory, institutional, and territorial dimensions to be operationalized and tested in subsequent papers.

A.6. Robustness and Falsifiability

The choices of functional form, horizon, cohorts, transformation, and thresholds are testable specifications, not necessary mathematical truths.

The construction will be evaluated according to four properties:

- transparency — components and parameters defined;
- reproducibility — calculation reproducible;
- robustness — stability in the face of alternative specifications;
- informational value — additional contribution relative to conventional indicators.

The planned analyses include different time horizons, functional forms, transformations, age groups, thresholds, and historical backtesting.

The objective is not to retrospectively select the specification that best produces a given result, but to verify how much of the conclusions depends on specific parameterization choices.

Summary: $N^*(i,t) = \sqrt{[NGII_puro(i,t) \times \text{Fator_Geracional}(i,t)]}$

NGII_puro provides the structural dimension; Fator_Geracional, the trajectory; and the transformation produces the final scale of the index.

NGII_puro measures the capacity for endogenous biological renewal of the population — not social configurations of care or transmission. This distinction is operationally necessary: social reproduction $\neq$ biological reproduction. A society may maintain robust social care arrangements while progressively losing the capacity to biologically renew its population — which NGII_puro captures and TFR in isolation does not capture with the same structural precision.

N is, therefore, a proposed and testable diagnostic instrument, not a causal assertion independent of empirical tests.*

N does not claim to be the only possible way of representing generational renewal. It claims that this form can be made explicit, calculated, compared, and subjected to the possibility of refutation.

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